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The research contributed to opening new horizons in topological spaces through a new definition of soft topological spaces, which researchers can use as a reference in their field of work

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ORIGINAL STUDY

Some Concepts Related to Supra Soft $\mathscr{v}-$ Open

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Abstract

This article introduce a new idea in the field of topological space which is supra soft $\mathscr{v}-$ open set and this concept is another generalization of a soft open set as well as the concept of supra soft $\mathscr{v}-$ closure is studied. Furthermore, the notion of supra soft $\mathscr{v}-$ interior is introduced and some properties of this concept were discussed. Finally, the concept of supra soft $\mathscr{v}-$ exterior is introduced and basic properties of this concept are investigated.

Keywords: Open set, Open, Interior point and closure

1. Introduction and basic concepts

General topology normally considers local properties of spaces, and is closely related to analysis.

The concept of supra topological space was proposed by Mashhour [1] in 1983 as a generalization of the concept of topological space.

Soft set theory is a tool for solving problems with uncertainty, the concept of soft set was first introduced by Molodtsov [2] in 1999.

The concept of soft topology was studied by Karata in 2011 [3].

The concept of supra soft topological space was studied by El-Sheikh and El-latif [4] in 2014 as a generalization of the concept of soft topological space.

The concept of an $\mathscr{v}-$ open set was first introduced in 2023 by Sameer and Abdalbaqi [5].

The concept of semi-open set was studied by Levine [6] in 1963 which is a generalization of an open set.

The notion of $\alpha -$ open set was studied in Ref. [7] as a generalization of open set, where $H \subseteq X$ is a $\alpha -$ open iff $H \subseteq (int(cl(int(H))))$.

The idea of $\beta -$ open set was studied in Ref. [8], where $H \subseteq X$ is a $\beta -$ open iff $H \subseteq (cl(int(cl(H))))$.

The main contribution in the spaces of supra soft topology are studying by Al-shami in 2019 [9,10] and 2022 [11].

In this paper we introduce and study the concept of supra soft $\mathscr{v}-$ open set another generalization of an open set.

Let X be a universal set and $\mathscr{E}$ is a set of parameters. If $\mathscr{S}_{\tilde{\mathscr{E}}}$ is a universal soft set and $\{\mathfrak{F}_\kappa\}_{\kappa \in J}$, $\kappa \geq 2$, be a collection of supra soft topologies on $\mathscr{S}_{\tilde{\mathscr{E}}}$ and $\mathscr{S}_{\mathscr{V}} \tilde{\subseteq} \mathscr{S}_{\tilde{\mathscr{E}}}$. Then $\mathscr{S}_{\mathscr{V}}$ is called supra soft $\mathscr{v}-$ open set in $\mathscr{S}_{\tilde{\mathscr{E}}}$ if there is $\mathscr{S}_T \in \bigcap_{\kappa \in J} \mathfrak{F}_\kappa$ such that $\mathscr{S}_\Phi \neq \mathscr{S}_T \tilde{\subseteq} \mathscr{S}_{\mathscr{V}}$ where $\mathscr{S}_{\mathscr{V}} \neq \mathscr{S}_\Phi$ and $\mathscr{S}_T = \mathscr{S}_\Phi$ where $\mathscr{S}_{\mathscr{V}} = \mathscr{S}_\Phi$. The set of all supra soft $\mathscr{v}-$ open in $\mathscr{S}_{\tilde{\mathscr{E}}}$ is denoted by $\mathscr{S}\mathscr{v}O_{\mathscr{S}_{\tilde{\mathscr{E}}}}$ and $(\mathscr{X}, \mathscr{S}\mathscr{v}O_{\mathscr{S}_{\tilde{\mathscr{E}}}}, \mathscr{E})$ is called supra soft $\mathscr{v}-$ space

The complement of a supra soft $\mathscr{v}-$ open set is called supra soft $\mathscr{v}-$ closed set and the set of all supra soft $\mathscr{v}-$ closed is denoted by $\mathscr{S}\mathscr{v}C_{\mathscr{S}_{\tilde{\mathscr{E}}}}$.

2. The main results

Definition (2.1). Let $(\mathscr{X}, \mathscr{S}\mathscr{v}O_{\mathscr{S}_{\tilde{\mathscr{E}}}}, \mathscr{E})$ be a supra soft $\mathscr{v}-$ space. A supra soft $\mathscr{v}-$ closure of $\mathscr{S}_{\mathscr{V}} \tilde{\subseteq} \mathscr{S}_{\tilde{\mathscr{E}}}$ is denoted by $cl_{\mathscr{S}_\mathscr{v}}(\mathscr{S}_{\mathscr{V}})$ and defined as the intersection of all supra soft $\mathscr{v}-$ closed sets that contains $\mathscr{S}_{\mathscr{V}}$.

Theorem (2.2). $cl_{\mathscr{S}_\mathscr{v}}(\mathscr{S}_{\mathscr{V}})$ is the smallest supra soft $\mathscr{v}-$ closed set that contain $\mathscr{S}_{\mathscr{V}}$.

Proof. An arbitrary intersection of supra soft $\mathscr{v}-$ closed sets is a supra soft $\mathscr{v}-$ closed, so we get $cl_{\mathscr{S}_\mathscr{v}}(\mathscr{S}_{\mathscr{V}})$ is a supra soft $\mathscr{v}-$ closed set.

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$\mathscr{S}_{\mathscr{V}} \tilde{\subseteq} cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}})$ by the definition of $cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}})$.
Let ${\mathscr{S}_{\mathscr{V}}}^*$ be a supra soft $\nu-$ closed set that contain $\mathscr{S}_{\mathscr{V}}$. Then ${\mathscr{S}_{\mathscr{V}}}^*$ includes the intersection of all supra soft $\nu-$ closed sets that contains $\mathscr{S}_{\mathscr{V}}$. Hence, $cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}}) \tilde{\subseteq} {\mathscr{S}_{\mathscr{V}}}^*$. This completes the proof.

Corollary (2.3). Suppose that $\mathscr{S}_{\mathscr{V}} \tilde{\subseteq} \mathscr{S}_{\tilde{\mathscr{E}}}$, then $\mathscr{S}_{\mathscr{V}}$ is a supra soft $\nu-$ closed if and only if $\mathscr{S}_{\mathscr{V}} = cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}})$.

Theorem (2.4). If $(\mathscr{X}, \mathscr{S}\nu O_{\mathscr{S}_{\tilde{\mathscr{E}}}}, \mathscr{E})$ is supra soft $\nu-$ space and $\mathscr{S}_{\mathscr{V}_1}, \mathscr{S}_{\mathscr{V}_2} \tilde{\subseteq} \mathscr{S}_{\tilde{\mathscr{E}}}$. Then

1. $\mathscr{S}_{\mathscr{V}_1} \tilde{\subseteq} \mathscr{S}_{\mathscr{V}_2}$, then $cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_1}) \tilde{\subseteq} cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_2})$.
2. $cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_1} \tilde{\bigcup} \mathscr{S}_{\mathscr{V}_2}) = cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_1}) \tilde{\bigcup} cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_2})$.
3. $cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_1} \tilde{\bigcap} \mathscr{S}_{\mathscr{V}_2}) \tilde{\subseteq} cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_1}) \tilde{\bigcap} cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_2})$.
4. $cl_{\mathscr{S}_\nu}(cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}})) = cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}})$, for any $\mathscr{S}_{\mathscr{V}} \tilde{\subseteq} \mathscr{S}_{\tilde{\mathscr{E}}}$.
5. $cl_{\mathscr{S}_\nu}(\mathscr{S}_\phi) = \mathscr{S}_\phi$ and $cl_{\mathscr{S}_\nu}(\mathscr{S}_{\tilde{\mathscr{E}}}) = \mathscr{S}_{\tilde{\mathscr{E}}}$.

Proof.

1. Since $cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_2})$ is a supra soft $\nu-$ closed set that contain $\mathscr{S}_{\mathscr{V}_2}$ and $\mathscr{S}_{\mathscr{V}_1} \tilde{\subseteq} \mathscr{S}_{\mathscr{V}_2}$, then $cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_2})$ is a supra soft $\nu-$ closed set that contain $\mathscr{S}_{\mathscr{V}_1}$, but $cl_{\mathscr{S}_\nu}(\mathscr{V}_1)$ is the smallest supra soft $\nu-$ closed that contain $\mathscr{S}_{\mathscr{V}_1}$, thus $cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_1}) \tilde{\subseteq} cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_2})$.
2. Since $\mathscr{S}_{\mathscr{V}_1} \tilde{\subseteq} \mathscr{S}_{\mathscr{V}_1} \tilde{\bigcup} \mathscr{S}_{\mathscr{V}_2}$ and $\mathscr{S}_{\mathscr{V}_2} \tilde{\subseteq} \mathscr{S}_{\mathscr{V}_1} \tilde{\bigcup} \mathscr{S}_{\mathscr{V}_2}$, then by [1] we get, $cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_1}) \tilde{\subseteq} cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_1} \tilde{\bigcup} \mathscr{S}_{\mathscr{V}_2})$ and $cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_2}) \tilde{\subseteq} cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_1} \tilde{\bigcup} \mathscr{S}_{\mathscr{V}_2})$. So, we have $cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_1}) \tilde{\bigcup} cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_2}) \tilde{\subseteq} cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_1} \tilde{\bigcup} \mathscr{S}_{\mathscr{V}_2})$.
   Now, $cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_1})$, $cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_2})$ are supra soft $\nu-$ closed sets that contains $\mathscr{S}_{\mathscr{V}_1}, \mathscr{S}_{\mathscr{V}_2}$ respectively, then $cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_1}) \tilde{\bigcup} cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_2})$ is a supra soft $\nu-$ closed set that contains $\mathscr{S}_{\mathscr{V}_1} \tilde{\bigcup} \mathscr{S}_{\mathscr{V}_2}$, but $cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_1} \tilde{\bigcup} \mathscr{S}_{\mathscr{V}_2})$ is the smallest supra soft $\nu-$ closed set that contain $\mathscr{S}_{\mathscr{V}_1} \tilde{\bigcup} \mathscr{S}_{\mathscr{V}_2}$, thus $cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_1} \tilde{\bigcup} \mathscr{S}_{\mathscr{V}_2}) \tilde{\subseteq} cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_1}) \tilde{\bigcup} cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_2})$. This is completes the proof.
3. Since $\mathscr{S}_{\mathscr{V}_1} \tilde{\bigcap} \mathscr{S}_{\mathscr{V}_2} \tilde{\subseteq} \mathscr{S}_{\mathscr{V}_1}$ and $\mathscr{S}_{\mathscr{V}_1} \tilde{\bigcap} \mathscr{S}_{\mathscr{V}_2} \tilde{\subseteq} \mathscr{S}_{\mathscr{V}_2}$, then by [1] we get, $cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_1} \tilde{\bigcap} \mathscr{S}_{\mathscr{V}_2}) \tilde{\subseteq} cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_1})$ and $cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_1} \tilde{\bigcap} \mathscr{S}_{\mathscr{V}_2}) \tilde{\subseteq} cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_2})$. So, we have $cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_1} \tilde{\bigcap} \mathscr{S}_{\mathscr{V}_2}) \tilde{\subseteq} cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_1}) \tilde{\bigcap} cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}_2})$.
4. Since $cl_{\mathscr{S}_\nu}(cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}}))$ is a supra soft $\nu-$ closed set that contain $cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}})$ and $\mathscr{S}_{\mathscr{V}} \tilde{\subseteq} cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}})$, then $cl_{\mathscr{S}_\nu}(cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}}))$ is a supra soft $\nu-$ closed that contain $\mathscr{S}_{\mathscr{V}}$. But $cl_{\mathscr{S}_\nu}(\mathscr{V})$ is the smallest supra soft $\nu-$ closed that contain $\mathscr{S}_{\mathscr{V}}$. Thus $cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}}) \tilde{\subseteq} cl_{\mathscr{S}_\nu}(cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}}))$. Clearly $cl_{\mathscr{S}_\nu}(cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}})) \tilde{\subseteq} cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}})$. Consequentially, $cl_{\mathscr{S}_\nu}(cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}})) = cl_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}})$.
5. Direct.

Definition (2.5). Let $(\mathscr{X}, \mathscr{S}\nu O_{\mathscr{S}_{\tilde{\mathscr{E}}}}, \mathscr{E})$ be a supra soft $\nu-$ space and $\mathscr{S}_{\mathscr{V}} \tilde{\subseteq} \mathscr{S}_{\tilde{\mathscr{E}}}$. A point $d \in \mathscr{X}$ is a supra soft $\nu-$ limit point of $\mathscr{S}_{\mathscr{V}}$ if $(\mathscr{S}_{\mathscr{M}} - \mathscr{S}_d) \tilde{\bigcap} \mathscr{S}_{\mathscr{V}} \neq \mathscr{S}_\phi$ $\forall \mathscr{S}_{\mathscr{M}} \in \mathscr{S}\nu O_{\mathscr{S}_{\tilde{\mathscr{E}}}}$ containing d.
The set of all supra soft $\nu-$ limit points of $\mathscr{S}_{\mathscr{V}}$ is denoted by $D_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}})$.

Theorem (2.6). Let $\mathscr{S}_{\mathscr{V}} \tilde{\subseteq} \mathscr{S}_{\tilde{\mathscr{E}}}$. Then $\mathscr{S}_{\mathscr{V}}$ is supra soft $\nu-$ closed if and only if $D_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}}) \tilde{\subseteq} \mathscr{S}_{\mathscr{V}}$.

Proof. Assume $\mathscr{S}_{\mathscr{V}}$ is supra soft $\nu-$ closed and $d \in D_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}})$. If $d \notin \mathscr{S}_{\mathscr{V}}$, then $d \in {\mathscr{S}_{\mathscr{V}}}^c$, but ${\mathscr{S}_{\mathscr{V}}}^c \in \mathscr{S}\nu O_{\mathscr{S}_{\tilde{\mathscr{E}}}}$, then $({\mathscr{S}_{\mathscr{V}}}^c - \mathscr{S}_d) \tilde{\bigcap} \mathscr{S}_{\mathscr{V}} = \mathscr{S}_\phi$, which implies that $d \notin D_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}})$ contradiction. Hence $d \in \mathscr{S}_{\mathscr{V}}$ and $D_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}}) \tilde{\subseteq} \mathscr{S}_{\mathscr{V}}$.
Conversely: suppose $D_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}}) \tilde{\subseteq} \mathscr{S}_{\mathscr{V}}$. To prove $\mathscr{S}_{\mathscr{V}}$ is supra soft $\nu-$ closed, we must prove ${\mathscr{S}_{\mathscr{V}}}^c \in \mathscr{S}\nu O_{\mathscr{S}_{\tilde{\mathscr{E}}}}$. Now, let $d \in {\mathscr{S}_{\mathscr{V}}}^c$, then $d \notin \mathscr{S}_{\mathscr{V}}$, hence $d \notin D_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}})$, thus $\exists\ \mathscr{S}_{\mathscr{M}} \in \mathscr{S}\nu O_{\mathscr{S}_{\tilde{\mathscr{E}}}}$ containing d s.t $(\mathscr{S}_{\mathscr{M}} - \mathscr{S}_d) \tilde{\bigcap} \mathscr{S}_{\mathscr{V}} = \mathscr{S}_\phi$.
Thus $\mathscr{S}_{\mathscr{M}} \tilde{\bigcap} \mathscr{S}_{\mathscr{V}} = \mathscr{S}_\phi$ since $d \notin \mathscr{S}_{\mathscr{V}}$. Consequentially, $\mathscr{S}_{\mathscr{M}} \tilde{\subseteq} {\mathscr{S}_{\mathscr{V}}}^c$. This completes the proof.

Theorem (2.7). Let $\mathscr{S}_{\mathscr{V}} \tilde{\subseteq} \mathscr{S}_{\tilde{\mathscr{E}}}$. Then $D_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}})$ is a supra soft $\nu-$ closed.

Proof. To prove $D_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}})$ is a supra soft $\nu-$ closed, we must prove $D_{\mathscr{S}_\nu}(D_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}})) \subseteq D_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}})$. Let $d \in D_{\mathscr{S}_\nu}(D_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}}))$. Then d is a supra soft $\nu -$ limit point of $D_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}})$. Hence $(\mathscr{S}_{\mathscr{M}} - \mathscr{S}_d) \tilde{\bigcap} D_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}}) \neq \phi$ $\forall \mathscr{S}_{\mathscr{M}} \in \mathscr{S}\nu O_{\mathscr{S}_{\tilde{\mathscr{E}}}}$ containing d, thus $(\mathscr{S}_{\mathscr{M}} - \mathscr{S}_d) \tilde{\bigcap} \mathscr{S}_{\mathscr{V}} \neq \mathscr{S}_\phi$ $\forall \mathscr{S}_{\mathscr{M}} \in \mathscr{S}\nu O_{\mathscr{S}_{\tilde{\mathscr{E}}}}$ containing d, which implies d is a supra soft $\nu-$ limit point of $\mathscr{S}_{\mathscr{V}}$ that is $d \in D_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}})$. Therefore $D_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}})$ is a supra soft $\nu-$ closed.

Theorem (2.8). If $\mathscr{S}_{\mathscr{V}} \tilde{\subseteq} \mathscr{S}_{\tilde{\mathscr{E}}}$. Then $\mathscr{S}_{\mathscr{V}} \tilde{\bigcup} D_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}})$ is a supra soft $\nu-$ closed.

Proof. To prove $\mathscr{S}_{\mathscr{V}} \tilde{\bigcup} D_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}})$ is a supra soft $\nu-$ closed, we must prove $(\mathscr{S}_{\mathscr{V}} \tilde{\bigcup} D_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}}))^c$ is a supra soft $\nu-$ open.
Let $d \in (\mathscr{S}_{\mathscr{V}} \tilde{\bigcup} D_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}}))^c$. Then $\notin \mathscr{S}_{\mathscr{V}} \tilde{\bigcup} D_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}})$, thus $d \notin \mathscr{S}_{\mathscr{V}}$ and $d \notin D_{\mathscr{S}_\nu}(\mathscr{S}_{\mathscr{V}})$. This implies that there is $\mathscr{S}_{\mathscr{M}} \in \mathscr{S}\nu O_{\mathscr{S}_{\tilde{\mathscr{E}}}}$ containing d such that $(\mathscr{S}_{\mathscr{M}} - \mathscr{S}_d) \tilde{\bigcap} \mathscr{S}_{\mathscr{V}} = \mathscr{S}_\phi$, but $d \notin \mathscr{S}_{\mathscr{V}}$, then $\mathscr{S}_{\mathscr{M}} \tilde{\bigcap} \mathscr{S}_{\mathscr{V}} = \mathscr{S}_\phi$.

We claim that $\mathscr{S}_{\mathscr{M}} \tilde{\bigcap} D_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}}) = \mathscr{S}_{\phi}$. Let $z \in \mathscr{S}_{\mathscr{M}}$, since $\mathscr{S}_{\mathscr{M}} \tilde{\bigcap} \mathscr{S}_{\mathscr{V}} = \mathscr{S}_{\phi}$, then $(\mathscr{S}_{\mathscr{M}} - \mathscr{S}_{z}) \tilde{\bigcap} \mathscr{S}_{\mathscr{V}} = \mathscr{S}_{\phi}$, hence $z \tilde{\notin} D_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}})$.
Thus $\mathscr{S}_{\mathscr{M}} \tilde{\bigcap} D_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}}) = \mathscr{S}_{\phi}$.
Now, $\mathscr{S}_{\mathscr{M}} \tilde{\bigcap} [\mathscr{S}_{\mathscr{V}} \tilde{\bigcup} D_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}})] = [\mathscr{S}_{\mathscr{M}} \tilde{\bigcap} \mathscr{V}] \tilde{\bigcup} [\mathscr{S}_{\mathscr{M}} \tilde{\bigcap} D_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}})] = \mathscr{S}_{\phi}$.
Then $\mathscr{S}_{\mathscr{M}} \tilde{\subseteq} [\mathscr{S}_{\mathscr{V}} \tilde{\bigcup} D_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}})]^{c}$. Hence proved.

Theorem (2.9). $cl_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}}) = \mathscr{S}_{\mathscr{V}} \tilde{\bigcup} D_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}})$ for any $\mathscr{S}_{\mathscr{V}} \tilde{\subseteq} \mathscr{S}_{\tilde{\mathscr{E}}}$.

Proof. From Theorem (2.8), we have $\mathscr{S}_{\mathscr{V}} \tilde{\bigcup} D_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}})$ is a supra soft $v-$ closed. Since $\mathscr{S}_{\mathscr{V}} \tilde{\subseteq} \mathscr{S}_{\mathscr{V}} \tilde{\bigcup} D_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}})$, but by Theorem (2.2), we have $cl_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}})$ is the smallest supra soft $v-$ closed that contain $\mathscr{S}_{\mathscr{V}}$. Then $cl_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}}) \tilde{\subseteq} \mathscr{S}_{\mathscr{V}} \tilde{\bigcup} D_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}})$. To prove $\mathscr{S}_{\mathscr{V}} \tilde{\bigcup} D_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}}) \tilde{\subseteq} cl_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}})$. Since $\mathscr{S}_{\mathscr{V}} \tilde{\subseteq} cl_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}})$, then it only remains to prove $D_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}}) \tilde{\subseteq} cl_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}})$, that is we must prove $D_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}}) \tilde{\subseteq} \widetilde{\bigcap_{i\in I}} \{\mathscr{S}_{\mathscr{V}_i} : \mathscr{S}_{\mathscr{V}_i}$ is supra soft $v-$ closed that contains $\mathscr{S}_{\mathscr{V}}\}$.
Let $d \in D_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}})$. Then $(\mathscr{S}_{\mathscr{M}} - \mathscr{S}_{d}) \tilde{\bigcap} \mathscr{S}_{\mathscr{V}} \neq \mathscr{S}_{\phi}$ $\forall \mathscr{S}_{\mathscr{M}} \in \mathscr{S}vO_{\mathscr{S}_{\tilde{\mathscr{E}}}}$ containing d. Hence $(\mathscr{S}_{\mathscr{M}} - \mathscr{S}_{d}) \tilde{\bigcap} \mathscr{S}_{\mathscr{V}_i} \neq \mathscr{S}_{\phi}$ $\forall \mathscr{S}_{\mathscr{M}} \in \mathscr{S}vO_{\mathscr{S}_{\tilde{\mathscr{E}}}}$ containing d, $\forall i \in$ I. Thus $d \in D_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}_i})$. But $\mathscr{S}_{\mathscr{V}_i}$ is supra soft $v-$ closed set $\forall i \in$I, then by Theorem (2.6), we have $\forall i \in$I, $D_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}_i}) \tilde{\subseteq} \mathscr{S}_{\mathscr{V}_i}$, hence $\forall i \in$I, $d \in \mathscr{S}_{\mathscr{V}_i} \tilde{\subseteq} \mathscr{S}_{\mathscr{V}}$.
Thus $d \in \widetilde{\bigcap_{i\in I}} \{\mathscr{S}_{\mathscr{V}_i} : \mathscr{S}_{\mathscr{V}_i}$ is supra soft $v-$ closed that contains $\mathscr{S}_{\mathscr{V}}\}$ that is $d \in cl_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}})$ which implies that $D_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}}) \tilde{\subseteq} cl_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}})$.
So, we have $\mathscr{S}_{\mathscr{V}} \tilde{\bigcup} D_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}}) \tilde{\subseteq} cl_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}})$.
Therefore, $cl_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}}) = \mathscr{S}_{\mathscr{V}} \tilde{\bigcup} D_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}})$.

Example (2.10). Let $\mathscr{X} = \{u_1, u_2, u_3\}$ and $\mathscr{E} = \{e_1, e_2\}$. Then

$$\tilde{P}(\mathscr{F}_{\tilde{\mathscr{E}}}) = \left\{ \begin{array}{c} \mathscr{S}_{\Phi}, \mathscr{S}_{\tilde{\mathscr{E}}}, \mathscr{S}_{\mathscr{V}_1}, \mathscr{S}_{\mathscr{V}_2}, \mathscr{S}_{\mathscr{V}_3}, \mathscr{S}_{\mathscr{V}_4}, \mathscr{S}_{\mathscr{V}_5}, \mathscr{S}_{\mathscr{V}_6}, \mathscr{S}_{\mathscr{V}_7}, \mathscr{S}_{\mathscr{V}_8}, \mathscr{S}_{\mathscr{V}_9}, \mathscr{S}_{\mathscr{V}_{10}}, \\ \mathscr{S}_{\mathscr{V}_{11}}, \mathscr{S}_{\mathscr{V}_{12}}, \mathscr{S}_{\mathscr{V}_{13}}, \mathscr{S}_{\mathscr{V}_{14}}, \mathscr{S}_{\mathscr{V}_{15}}, \mathscr{S}_{\mathscr{V}_{16}}, \mathscr{S}_{\mathscr{V}_{17}}, \mathscr{S}_{\mathscr{V}_{18}}, \mathscr{S}_{\mathscr{V}_{19}}, \mathscr{S}_{\mathscr{V}_{20}}, \\ \mathscr{S}_{\mathscr{V}_{21}}, \mathscr{S}_{\mathscr{V}_{22}}, \mathscr{S}_{\mathscr{V}_{23}}, \mathscr{S}_{\mathscr{V}_{24}}, \mathscr{S}_{\mathscr{V}_{25}}, \mathscr{S}_{\mathscr{V}_{26}}, \mathscr{S}_{\mathscr{V}_{27}}, \mathscr{S}_{\mathscr{V}_{28}}, \mathscr{S}_{\mathscr{V}_{29}}, \mathscr{S}_{\mathscr{V}_{30}}, \\ \mathscr{S}_{\mathscr{V}_{31}}, \mathscr{S}_{\mathscr{V}_{32}}, \mathscr{S}_{\mathscr{V}_{33}}, \mathscr{S}_{\mathscr{V}_{34}}, \mathscr{S}_{\mathscr{V}_{35}}, \mathscr{S}_{\mathscr{V}_{36}}, \mathscr{S}_{\mathscr{V}_{37}}, \mathscr{S}_{\mathscr{V}_{38}}, \mathscr{S}_{\mathscr{V}_{39}}, \mathscr{S}_{\mathscr{V}_{40}}, \\ \mathscr{S}_{\mathscr{V}_{41}}, \mathscr{S}_{\mathscr{V}_{42}}, \mathscr{S}_{\mathscr{V}_{43}}, \mathscr{S}_{\mathscr{V}_{44}}, \mathscr{S}_{\mathscr{V}_{45}}, \mathscr{S}_{\mathscr{V}_{46}}, \mathscr{S}_{\mathscr{V}_{47}}, \mathscr{S}_{\mathscr{V}_{48}}, \mathscr{S}_{\mathscr{V}_{49}}, \mathscr{S}_{\mathscr{V}_{50}}, \mathscr{S}_{\mathscr{V}_{51}}, \\ \mathscr{S}_{\mathscr{V}_{52}}, \mathscr{S}_{\mathscr{V}_{53}}, \mathscr{S}_{\mathscr{V}_{54}}, \mathscr{S}_{\mathscr{V}_{55}}, \mathscr{S}_{\mathscr{V}_{56}}, \mathscr{S}_{\mathscr{V}_{57}}, \mathscr{S}_{\mathscr{V}_{58}}, \mathscr{S}_{\mathscr{V}_{59}}, \mathscr{S}_{\mathscr{V}_{60}}, \mathscr{S}_{\mathscr{V}_{61}}, \mathscr{S}_{\mathscr{V}_{62}} \end{array} \right\}$$

Where:

$\mathscr{S}_{\mathscr{V}_1} = \{(e_1, \{u_1\}), (e_2, \{u_2\})\}$,

$\mathscr{S}_{\mathscr{V}_2} = \{(e_1, \{u_1\}), (e_2, \{u_1, u_2\})\}$,

$\mathscr{S}_{\mathscr{V}_3} = \{(e_1, \{u_1\}), (e_2, \{u_2, u_3\})\}$,

$\mathscr{S}_{\mathscr{V}_4} = \{(e_1, \{u_1\}), (e_2, \mathscr{X})\}$

$\mathscr{S}_{\mathscr{V}_5} = \{(e_1, \{u_1, u_2\}), (e_2, \{u_2\})\}$,

$\mathscr{S}_{\mathscr{V}_6} = \{(e_1, \{u_1, u_3\}), (e_2, \{u_2\})\}$

$\mathscr{S}_{\mathscr{V}_7} = \{(e_1, \mathscr{X}), (e_2, \{u_2\})\}$,

$\mathscr{S}_{\mathscr{V}_8} = \{(e_1, \{u_1, u_2\}), (e_2, \{u_1, u_2\})\}$,

$\mathscr{S}_{\mathscr{V}_9} = \{(e_1, \{u_1, u_2\}), (e_2, \{u_2, u_3\})\}$,

$\mathscr{S}_{\mathscr{V}_{10}} = \{(e_1, \{u_1, u_3\}), (e_2, \{u_1, u_2\})\}$,

$\mathscr{S}_{\mathscr{V}_{11}} = \{(e_1, \{u_1, u_3\}), (e_2, \{u_2, u_3\})\}$,

$\mathscr{S}_{\mathscr{V}_{12}} = \{(e_1, \{u_1, u_2\}), (e_2, \mathscr{X})\}$,

$\mathscr{S}_{\mathscr{V}_{13}} = \{(e_1, \{u_1, u_3\}), (e_2, \mathscr{X})\}$,

$\mathscr{S}_{\mathscr{V}_{14}} = \{(e_1, \mathscr{X}), (e_2, \{u_2, u_3\})\}$,

$\mathscr{S}_{\mathscr{V}_{15}} = \{(e_1, \mathscr{X}), (e_2, \{u_1, u_2\})\}$,

$\mathscr{S}_{\mathscr{V}_{16}} = \{(e_1, \{u_2, u_3\}), (e_2, \{u_1, u_3\})\}$,

$\mathscr{S}_{\mathscr{V}_{17}} = \{(e_1, \{u_2, u_3\}), (e_2, \{u_3\})\}$

$\mathscr{S}_{\mathscr{V}_{18}} = \{(e_1, \{u_2, u_3\}), (e_2, \{u_1\})\}$,

$\mathscr{S}_{\mathscr{V}_{19}} = \{(e_1, \{u_2, u_3\})\}$,

$\mathscr{S}_{\mathscr{V}_{20}} = \{(e_1, \{u_3\}), (e_2, \{u_1, u_3\})\}$,

$\mathscr{S}_{\mathscr{V}_{21}} = \{(e_1, \{u_2\}), (e_2, \{u_1, u_3\})\}$

$\mathscr{S}_{\mathscr{V}_{22}} = \{(e_2, \{u_1, u_3\})\}$,

$\mathscr{S}_{\mathscr{V}_{23}} = \{(e_1, \{u_3\}), (e_2, \{u_3\})\}$,

$\mathscr{S}_{\mathscr{V}_{24}} = \{(e_1, \{u_3\}), (e_2, \{u_1\})\}$,

$\mathscr{S}_{\mathscr{V}_{25}} = \{(e_1, \{u_2\}), (e_2, \{u_3\})\}$,

$\mathscr{S}_{\mathscr{V}_{26}} = \{(e_1, \{u_2\}), (e_2, \{u_1\})\}$,

$\mathscr{S}_{\mathscr{V}_{27}} = \{(e_1, \{u_3\})\}$,

$\mathscr{S}_{\mathscr{V}_{28}} = \{(e_1, \{u_2\})\}$,

$\mathscr{S}_{\mathscr{V}_{29}} = \{(e_2, \{u_1\})\}$,

$\mathscr{S}_{\mathscr{V}_{30}} = \{(e_2, \{u_3\})\}$,

$\mathscr{S}_{\mathscr{V}_{31}} = \{(e_1, \{u_1\})\}$

$\mathscr{S}_{\mathscr{V}_{32}} = \{(e_2, \{u_2\})\}$

$\mathscr{S}_{\mathscr{V}_{33}} = \{(e_1, \{u_1, u_2\})\}$

$\mathscr{S}_{\mathscr{V}_{34}} = \{(e_1, \{u_1, u_3\})\}$

$\mathscr{S}_{\mathscr{V}_{35}} = \{(e_2, \{u_1, u_2\})\}$

$\mathscr{S}_{\mathscr{V}_{36}} = \{(e_2, \{u_2, u_3\})\}$

$\mathscr{S}_{\mathscr{V}_{37}} = \{(e_1, \mathscr{X})\}$

$\mathscr{S}_{\mathscr{V}_{38}} = \{(e_2, \mathscr{X})\}$

$\mathscr{S}_{\mathscr{V}_{39}} = \{(e_1, \{u_1\}), (e_2, \{u_1\})\}$,

$\mathscr{S}_{\mathscr{V}_{40}} = \{(e_1, \{u_1\}), (e_2, \{u_3\})\}$

$\mathscr{S}_{\mathscr{V}_{41}} = \{(e_1, \{u_1, u_2\}), (e_2, \{u_3\})\}$,

$\mathscr{S}_{\mathscr{V}_{42}} = \{(e_1, \{u_2\}), (e_2, \{u_3\})\}$

$\mathscr{S}_{\mathscr{V}_{43}} = \{(e_1, \{u_2\}), (e_2, \{u_2\})\}$,

$\mathscr{S}_{\mathscr{V}_{44}} = \{(e_1, \{u_3\}), (e_2, \{u_1, u_2\})\}$

$\mathscr{S}_{\mathscr{V}_{45}} = \{(e_1, \{u_3\}), (e_2, \{u_2\})\}$,

$\mathscr{S}_{\mathscr{V}_{46}} = \{(e_1, \{u_2\}), (e_2, \{u_1, u_2\})\}$

$\mathscr{S}_{\mathscr{V}_{47}} = \{(e_1, \{u_2\}), (e_2, \{u_2, u_3\})\}$

$\mathscr{S}_{\mathscr{V}_{48}} = \{(e_1, \{u_1, u_2\}), (e_2, \{u_1, u_3\})\}$

$\mathscr{S}_{\mathscr{V}_{49}} = \{(e_1, \{u_1, u_3\}), (e_2, \{u_1, u_3\})\}$

$\mathscr{S}_{\mathscr{V}_{50}} = \{(e_1, \{u_2, u_3\}), (e_2, \{u_1, u_2\})\}$

$\mathscr{S}_{\mathscr{V}_{51}} = \{(e_1, \{u_2, u_3\}), (e_2, \{u_2, u_3\})\}$

$\mathscr{S}_{\mathscr{V}_{52}} = \{(e_1, \{u_2, u_3\}), (e_2, \{u_2\})\}$

$\mathscr{S}_{\mathscr{V}_{53}} = \{(e_1, \{u_2\}), (e_2, \{u_2, u_3\})\}$

$\mathscr{S}_{\mathscr{V}_{54}} = \{(e_1, \{u_3\}), (e_2, \{u_2, u_3\})\}$

$\mathscr{S}_{\mathscr{V}_{55}} = \{(e_1, \{u_1, u_2\}), (e_2, \{u_1\})\}$

$\mathscr{S}_{\mathscr{V}_{56}} = \{(e_1, \{u_1, u_3\}), (e_2, \{u_1\})\}$

$\mathscr{S}_{\mathscr{V}_{57}} = \{(e_1, \mathscr{X}), (e_2, \{u_1\})\}$

$\mathscr{S}_{\mathscr{V}_{58}} = \{(e_1, \mathscr{X}), (e_2, \{u_3\})\}$

$\mathscr{S}_{\mathscr{V}_{59}} = \{(e_1, \mathscr{X}), (e_2, \{u_1, u_3\})\}$

$\mathscr{S}_{\mathscr{V}_{60}} = \{(e_1, \{u_2\}), (e_2, \mathscr{X})\}$

$\mathscr{S}_{\mathscr{V}_{61}} = \{(e_1, \{u_3\}), (e_2, \mathscr{X})\}$

$\mathscr{S}_{\mathscr{V}_{62}} = \{(e_1, \{u_2, u_3\}), (e_2, \mathscr{X})\}$

Define $\mathfrak{F}_1$, $\mathfrak{F}_2$, $\mathfrak{F}_3$ and $\mathfrak{F}_4$ as follows:

$\mathfrak{F}_1 = \{\mathscr{S}_\Phi, \mathscr{S}_{\tilde{\mathscr{E}}}, \mathscr{S}_{\mathscr{V}_1}, \mathscr{S}_{\mathscr{V}_9}\}$

$\mathfrak{F}_2 = \{\mathscr{S}_\Phi, \mathscr{S}_{\tilde{\mathscr{E}}}, \mathscr{S}_{\mathscr{V}_1}, \mathscr{S}_{\mathscr{V}_{48}}, \mathscr{S}_{\mathscr{V}_{12}}\}$

$\mathfrak{F}_3 = \{\mathscr{S}_\Phi, \mathscr{S}_{\tilde{\mathscr{E}}}, \mathscr{S}_{\mathscr{V}_1}, \mathscr{S}_{\mathscr{V}_{49}}, \mathscr{S}_{\mathscr{V}_{13}}\}$

$\mathfrak{F}_4 = \{\mathscr{S}_\Phi, \mathscr{S}_{\tilde{\mathscr{E}}}, \mathscr{S}_{\mathscr{V}_1}, \mathscr{S}_{\mathscr{V}_{50}}, \mathscr{S}_{\mathscr{V}_{15}}\}$

Then $(\mathscr{X}, \mathfrak{F}_1, \mathscr{E})$, $(\mathscr{X}, \mathfrak{F}_2, \mathscr{E})$ are supra soft spaces. Now, $\bigcap_{\kappa=1}^{4} \mathfrak{F}_\kappa = \{\mathscr{S}_\Phi, \mathscr{S}_{\tilde{\mathscr{E}}}, \mathscr{S}_{\mathscr{V}_1}\}$, so, we note that $\mathscr{S}_{\mathscr{V}_1} \in \bigcap_{\kappa=1}^{4} \mathfrak{F}_\kappa$ and $\mathscr{S}_{\mathscr{V}_1} \tilde{\subseteq} \mathscr{S}_{\mathscr{V}_i}$ *for all* $i = 1, 2, \ldots, 15$, then $\mathscr{S}_{\mathscr{V}_i}$ for all i=1,2,…,15 are supra soft $\mathscr{o}-$ open It is clear that $\mathscr{S}_\Phi$ & $\mathscr{S}_{\tilde{\mathscr{E}}}$ supra soft $\mathscr{o}-$ open.
In the other hand, we note that $\mathscr{S}_{\mathscr{V}_{31}}$, $\mathscr{S}_{\mathscr{V}_{32}}$, …, $\mathscr{S}_{\mathscr{V}_{62}}$ are not supra soft $\mathscr{o}-$ open, because there is no $\mathscr{S}_T \in \bigcap_{\kappa=1}^{4} \mathfrak{F}_\kappa$ such that $\mathscr{S}_T \neq \mathscr{S}_\Phi$ and $\mathscr{S}_T \tilde{\subseteq} \mathscr{S}_{\mathscr{V}_i}, i = 31, 32, \ldots, 62$. Thus,

$$\mathscr{S}\mathscr{o}O_{\mathscr{S}_{\tilde{\mathscr{E}}}} = \left\{ \begin{array}{l} \mathscr{S}_\Phi, \mathscr{S}_{\tilde{\mathscr{E}}}, \mathscr{S}_{\mathscr{V}_1}, \mathscr{S}_{\mathscr{V}_2}, \mathscr{S}_{\mathscr{V}_3}, \mathscr{S}_{\mathscr{V}_4}, \mathscr{S}_{\mathscr{V}_5}, \mathscr{S}_{\mathscr{V}_6}, \mathscr{S}_{\mathscr{V}_7}, \\ \mathscr{S}_{\mathscr{V}_8}, \mathscr{S}_{\mathscr{V}_9}, \mathscr{S}_{\mathscr{V}_{10}}, \mathscr{S}_{\mathscr{V}_{11}}, \mathscr{S}_{\mathscr{V}_{12}}, \mathscr{S}_{\mathscr{V}_{13}}, \mathscr{S}_{\mathscr{V}_{14}}, \mathscr{S}_{\mathscr{V}_{15}} \end{array} \right\}$$

and

$$\mathscr{S}v\mathrm{C}_{\mathscr{S}_{\tilde{\mathscr{E}}}} = \left\{ \begin{array}{c} \mathscr{S}_{\Phi}, \mathscr{S}_{\tilde{\mathscr{E}}}, \mathscr{S}_{\mathscr{V}_{16}}, \mathscr{S}_{\mathscr{V}_{17}}, \mathscr{S}_{\mathscr{V}_{18}}, \mathscr{S}_{\mathscr{V}_{19}}, \mathscr{S}_{\mathscr{V}_{20}}, \mathscr{S}_{\mathscr{V}_{21}}, \mathscr{S}_{\mathscr{V}_{22}}, \\ \mathscr{S}_{\mathscr{V}_{23}}, \mathscr{S}_{\mathscr{V}_{24}}, \mathscr{S}_{\mathscr{V}_{25}}, \mathscr{S}_{\mathscr{V}_{26}}, \mathscr{S}_{\mathscr{V}_{27}}, \mathscr{S}_{\mathscr{V}_{28}}, \mathscr{S}_{\mathscr{V}_{29}}, \mathscr{S}_{\mathscr{V}_{30}} \end{array} \right\}$$

Therefore, $(\mathscr{X}, \mathscr{S}v\mathrm{O}_{\mathscr{S}_{\tilde{\mathscr{E}}}}, \mathscr{E})$ is a supra soft $v-$ space. Now, $cl_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}_{16}}) = \mathscr{S}_{\mathscr{V}_{16}}$ and $cl_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{V}_{32}}) = \mathscr{S}_{\tilde{\mathscr{E}}}$

Proposition (2.11). Let $\mathscr{X}$ be a universal set and $\mathscr{E}$ is a set of parameters with respect to $\mathscr{X}$ and let $\{\mathfrak{F}_\kappa\}_{\kappa\in J}$, $\kappa \geq 2$, be a collection of supra soft topologies on $\mathscr{S}_{\tilde{\mathscr{E}}}$. If $\mathscr{S}_{\mathrm{T}} \in \mathfrak{F}_\kappa$ for all $\kappa \in J$, then

$$cl^{\mathfrak{F}_\kappa}(\mathscr{S}_{\mathrm{T}}{}^c) = cl_{\mathscr{S}_v}(\mathscr{S}_{\mathrm{T}}{}^c) = \mathscr{S}_{\mathrm{T}}{}^c.$$

Proof. Assume that $\mathscr{S}_{\mathrm{T}} \in \mathfrak{F}_\kappa$ for all $\kappa \in J$, then we have $\mathscr{S}_{\mathrm{T}} \in \mathscr{S}v\mathrm{O}_{\mathscr{S}_{\tilde{\mathscr{E}}}}$, thus $\mathscr{S}_{\mathrm{T}}$ is a supra soft $v-$ open, therefore $\mathscr{S}_{\mathrm{T}}{}^c$ is a supra soft $v-$ closed, so by Corollary (2.3), we have $cl_{\mathscr{S}_v}(\mathscr{S}_{\mathrm{T}}{}^c) = \mathscr{S}_{\mathrm{T}}{}^c$.
Now, $\mathscr{S}_{\mathrm{T}} \in \mathfrak{F}_\kappa$ for all $\kappa \in J$, then $\mathscr{S}_{\mathrm{T}}$ is a supra soft open set in $(\mathscr{X}, \mathfrak{F}_\kappa, \mathscr{E})$ for all $\kappa \in J$, that is $\mathscr{S}_{\mathrm{T}}{}^c$ is a supra soft closed in $(\mathscr{X}, \mathfrak{F}_\kappa, \mathscr{E})$ for all $\kappa \in J$, thus $cl^{\mathfrak{F}_\kappa}(\mathscr{S}_{\mathrm{T}}{}^c) = \mathscr{S}_{\mathrm{T}}{}^c$.
Consequentially, $cl^{\mathfrak{F}_\kappa}(\mathscr{S}_{\mathrm{T}}{}^c) = cl_{\mathscr{S}_v}(\mathscr{S}_{\mathrm{T}}{}^c) = \mathscr{S}_{\mathrm{T}}{}^c$.

Corollary (2.12). Let $\mathscr{X}$ be a universal set and $\mathscr{E}$ is a set of parameters with respect to $\mathscr{X}$ and let $\{\mathfrak{F}_\kappa\}_{\kappa\in J}$, $\kappa \geq 2$, be a collection of supra soft topologies on $\mathscr{S}_{\tilde{\mathscr{E}}}$. If $\mathscr{S}_{\mathrm{T}} \in \bigcap_{\kappa\in J} \mathfrak{F}_\kappa$.
Then $cl^{\bigcap_{\kappa\in J}\mathfrak{F}_\kappa}(\mathscr{S}_{\mathrm{T}}{}^c) = \mathscr{S}_{\mathrm{T}}{}^c = cl_{\mathscr{S}_v}(\mathscr{S}_{\mathrm{T}}{}^c)$.

Proof. Assume that $\mathscr{S}_{\mathrm{T}} \in \bigcap_{\kappa\in J} \mathfrak{F}_\kappa$, then $\mathscr{S}_{\mathrm{T}} \in \mathfrak{F}_\kappa$ for all $\kappa \in J$, hence by Proposition (2.11), we have $cl^{\mathfrak{F}_\kappa}(\mathscr{S}_{\mathrm{T}}{}^c) = \mathscr{S}_{\mathrm{T}}{}^c = cl_{\mathscr{S}_v}(\mathscr{S}_{\mathrm{T}}{}^c)$.
But, $\mathscr{S}_{\mathrm{T}}{}^c$ is a supra soft closed in $(\mathscr{X}, \mathfrak{F}_\kappa, \mathscr{E})$ for all $\kappa \in J$, then $\mathscr{S}_{\mathrm{T}}{}^c$ is a supra soft closed in $(\mathscr{X}, \bigcap_{\kappa\in J} \mathfrak{F}_\kappa, \mathscr{E})$.
Therefore, $cl^{\bigcap_{\kappa\in J}\mathfrak{F}_\kappa}(\mathscr{S}_{\mathrm{T}}{}^c) = \mathscr{S}_{\mathrm{T}}{}^c = cl_{\mathscr{S}_v}(\mathscr{S}_{\mathrm{T}}{}^c)$.

Definition (2.13). Let $(\mathscr{X}, \mathscr{S}v\mathrm{O}_{\mathscr{S}_{\tilde{\mathscr{E}}}}, \mathscr{E})$ be a supra soft $v-$ space and $\mathscr{S}_{\mathscr{M}} \tilde{\subseteq} \mathscr{S}_{\tilde{\mathscr{E}}}$. A point $d \in \mathscr{S}_{\mathscr{M}}$ is called an supra soft $v-$ interior point of $\mathscr{S}_{\mathscr{M}}$ if there is $\mathscr{S}_{\mathscr{V}} \in \mathscr{S}v\mathrm{O}_{\mathscr{S}_{\tilde{\mathscr{E}}}}$ such that $\in \mathscr{S}_{\mathscr{V}} \tilde{\subseteq} \mathscr{S}_{\mathscr{M}}$.

Definition (2.14). Let $(\mathscr{X}, \mathscr{S}v\mathrm{O}_{\mathscr{S}_{\tilde{\mathscr{E}}}}, \mathscr{E})$ be a supra soft $v-$ space and $\mathscr{S}_{\mathscr{M}} \tilde{\subseteq} \mathscr{S}_{\tilde{\mathscr{E}}}$. The set of all supra soft $v-$ interior points of $\mathscr{S}_{\mathscr{M}}$ is called supra *soft* $v-$ interior of $\mathscr{S}_{\mathscr{M}}$ and is denoted by $lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}})$.

Theorem (2.15).

$$lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}}) = \widetilde{\bigcup_{i\in I}} \left\{ \begin{array}{c} \mathscr{S}_{\mathscr{V}_i} : \mathscr{S}_{\mathscr{V}_i} \text{ is supra soft } v\text{-open} \\ \text{which contained in } \mathscr{S}_{\mathscr{M}} \end{array} \right\}$$

Proof. Assume $d \in lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}})$, then d is an supra soft $v-$ interior point of $\mathscr{S}_{\mathscr{M}}$, hence there is $\mathscr{S}_{\mathscr{V}} \in \mathscr{S}v\mathrm{O}_{\mathscr{S}_{\tilde{\mathscr{E}}}}$ such that $d \in \mathscr{S}_{\mathscr{V}} \tilde{\subseteq} \mathscr{S}_{\mathscr{M}}$. Now, $\mathscr{S}_{\mathscr{V}}$ is a supra soft $v-$ open such that $\mathscr{S}_{\mathscr{V}} \tilde{\subseteq} \mathscr{S}_{\mathscr{M}}$. So, we have

$$d \in \widetilde{\bigcup_{i\in I}} \left\{ \begin{array}{c} \mathscr{S}_{\mathscr{V}_i} : \mathscr{S}_{\mathscr{V}_i} \text{ is supra soft } v\text{-open} \\ \text{which contained in } \mathscr{S}_{\mathscr{M}} \end{array} \right\}.$$

Which is implies that,

$$lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}}) \tilde{\triangleleft} \widetilde{\bigcup_{i\in I}} \left\{ \begin{array}{c} \mathscr{S}_{\mathscr{V}_i} : \mathscr{S}_{\mathscr{V}_i} \text{ is supra soft } v\text{-open} \\ \text{which contained in } \mathscr{S}_{\mathscr{M}} \end{array} \right\}.$$

Now, let

$$d \in \widetilde{\bigcup_{i\in I}} \left\{ \begin{array}{c} \mathscr{S}_{\mathscr{V}_i} : \mathscr{S}_{\mathscr{V}_i} \text{ is supra soft } v\text{-open} \\ \text{which contained in } \mathscr{S}_{\mathscr{M}} \end{array} \right\}.$$

Then there is supra soft $v-$ open $\mathscr{S}_{\mathscr{V}}{}^*$, $\mathscr{S}_{\mathscr{V}}{}^* \tilde{\subseteq} \mathscr{S}_{\mathscr{M}}$ such that $d \in \mathscr{S}_{\mathscr{V}}{}^*$, hence $d \in \mathscr{S}_{\mathscr{M}}$ is an supra soft $v-$ interior point of $\mathscr{S}_{\mathscr{M}}$, that is $d \in lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}})$. Thus $lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}}) \tilde{\supseteq}$

$$\widetilde{\bigcup_{i\in I}} \left\{ \begin{array}{c} \mathscr{S}_{\mathscr{V}_i} : \mathscr{S}_{\mathscr{V}_i} \text{ is supra soft } v\text{-open} \\ \text{which contained in } \mathscr{S}_{\mathscr{M}} \end{array} \right\}.$$

This completes the proof.

Theorem (2.16). Let $(\mathscr{X}, \mathscr{S}v\mathrm{O}_{\mathscr{S}_{\tilde{\mathscr{E}}}}, \mathscr{E})$ be a supra soft $v-$ space and $\mathscr{S}_{\mathscr{M}} \tilde{\subseteq} \mathscr{S}_{\tilde{\mathscr{E}}}$. Then $lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}})$ is the largest supra soft $v-$ open set contained in $\mathscr{S}_{\mathscr{M}}$.

Proof. From Theorem (2.15), we have

$$lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}}) = \widetilde{\bigcup_{i\in I}} \left\{ \begin{array}{c} \mathscr{S}_{\mathscr{V}_i} : \mathscr{S}_{\mathscr{V}_i} \text{ is supra soft } v\text{-open} \\ \text{which contained in } \mathscr{S}_{\mathscr{M}} \end{array} \right\}.$$

The arbitrary union of supra soft $v-$ open is also supra soft $v-$ open. Hence $lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}})$ is a supra soft $v-$ open set.

Let $\mathscr{S}_{\mathscr{V}_i}$ is supra soft $\mathscr{v}-$ open and $\mathscr{S}_{\mathscr{V}_i}\tilde{\subseteq}\mathscr{S}_{\mathscr{M}}$ $\forall i\in$ I. Then

$$\widetilde{\bigcup_{i\in I}}\left\{\begin{matrix}\mathscr{S}_{\mathscr{V}_i}:\mathscr{S}_{\mathscr{V}_i}\text{ is supra soft }\mathscr{v}-\text{open}\\ \text{which contained in }\mathscr{S}_{\mathscr{M}}\end{matrix}\right\}\tilde{\trianglelefteq}\mathscr{S}_{\mathscr{M}}.$$

Hence, $lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}})\tilde{\subseteq}\mathscr{S}_{\mathscr{M}}$.

Now, let $\mathscr{S}_{\mathscr{V}_i}{}^{*}$ be a supra soft $\mathscr{v}-$ open such that $\mathscr{S}_{\mathscr{V}_i}{}^{*}\subseteq\mathscr{S}_{\mathscr{M}}$. Then

$$\mathscr{S}_{\mathscr{V}_i}{}^{*}\tilde{\trianglelefteq}\widetilde{\bigcup_{i\in I}}\left\{\begin{matrix}\mathscr{S}_{\mathscr{V}_i}:\mathscr{S}_{\mathscr{V}_i}\text{ is supra soft }\mathscr{v}-\text{open}\\ \text{which contained in }\mathscr{S}_{\mathscr{M}}\end{matrix}\right\},$$

that is, $\mathscr{S}_{\mathscr{V}_i}{}^{*}\tilde{\subseteq}lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}})$.

Therefore, $lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}})$ is the largest supra soft $\mathscr{v}-$ open contained in $\mathscr{S}_{\mathscr{M}}$.

Theorem (2.17). $\mathscr{S}_{\mathscr{M}}$ is a supra soft $\mathscr{v}-$ open if and only if $lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}})=\mathscr{S}_{\mathscr{M}}$.

Proof. Assume $\mathscr{S}_{\mathscr{M}}$ is a supra soft $\mathscr{v}-$ open. From Theorem (2.16), we have $lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}})\tilde{\subseteq}\mathscr{S}_{\mathscr{M}}$. But $\mathscr{S}_{\mathscr{M}}$ is a supra soft $\mathscr{v}-$ open & $\mathscr{S}_{\mathscr{M}}\tilde{\subseteq}\mathscr{S}_{\mathscr{M}}$ and $lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}})$ is the largest supra soft $\mathscr{v}-$ open set contained in $\mathscr{S}_{\mathscr{M}}$. Then $\mathscr{S}_{\mathscr{M}}\tilde{\subseteq}lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}})$. Hence $lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}})=\mathscr{S}_{\mathscr{M}}$.

Conversely: suppose that $ln\,t_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}})=\mathscr{S}_{\mathscr{M}}$. By Theorem (2.16), we have $lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}})$ is a supra soft $\mathscr{v}-$ open set. This completes the proof.

Theorem (2.18). Let $(\mathscr{X},\mathscr{S}\mathscr{v}O_{\mathscr{S}_{\tilde{\mathscr{E}}}},\mathscr{E})$ be supra soft $\mathscr{v}-$ space and $\mathscr{S}_{\mathscr{M}},\mathscr{S}_{\mathscr{N}}\tilde{\subseteq}\mathscr{S}_{\tilde{\mathscr{E}}}$. Then

1. If $\mathscr{S}_{\mathscr{M}}\tilde{\subseteq}\mathscr{S}_{\mathscr{N}}$, then $lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}})\tilde{\subseteq}lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{N}})$.
2. $lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}}\tilde{\bigcap}\mathscr{S}_{\mathscr{N}})\tilde{\subseteq}lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}})\tilde{\bigcap}lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{N}})$.
3. $lnt_{\mathscr{S}_v}(lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}}))=lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}})$.
4. $lnt_{\mathscr{S}_v}(\mathscr{S}_{\phi})=\mathscr{S}_{\phi}$ and $lnt_{\mathscr{S}_v}(\mathscr{S}_{\tilde{\mathscr{E}}})=\mathscr{S}_{\tilde{\mathscr{E}}}$.

Proof.

1. Suppose that $\mathscr{S}_{\mathscr{M}}\tilde{\subseteq}\mathscr{S}_{\mathscr{N}}$. Since $lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}})$ is a supra soft $\mathscr{v}-$ open set contained in $\mathscr{S}_{\mathscr{M}}$, then $lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}})$ is a supra soft $\mathscr{v}-$ open set contained in $\mathscr{S}_{\mathscr{N}}$. But $lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{N}})$ is the largest supra soft $\mathscr{v}-$ open set contained in $\mathscr{S}_{\mathscr{N}}$, which implies to $lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}})\tilde{\subseteq}lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{N}})$.
2. Let $\mathscr{d}\in lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}}\tilde{\bigcap}\mathscr{S}_{\mathscr{N}})$, then $\mathscr{d}$ is an supra soft $\mathscr{v}-$ interior point of $\mathscr{S}_{\mathscr{M}}\tilde{\bigcap}\mathscr{S}_{\mathscr{N}}$, hence there is $\mathscr{S}_{\mathscr{V}}\in\mathscr{S}\mathscr{v}O_{\mathscr{S}_{\tilde{\mathscr{E}}}}$ such that $\mathscr{d}\in\mathscr{S}_{\mathscr{V}}\tilde{\subseteq}\mathscr{S}_{\mathscr{M}}\tilde{\bigcap}\mathscr{S}_{\mathscr{N}}$. Thus $\mathscr{d}\in\mathscr{S}_{\mathscr{V}}\tilde{\subseteq}\mathscr{S}_{\mathscr{M}}$ and $\in\mathscr{S}_{\mathscr{V}}\tilde{\subseteq}\mathscr{S}_{\mathscr{N}}$, therefore $\mathscr{d}$ is an supra soft $\mathscr{v}-$ interior point of $\mathscr{S}_{\mathscr{M}}$ *and* $\mathscr{S}_{\mathscr{N}}$. Thus $\mathscr{d}\in lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}})$ and $\mathscr{d}\in lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{N}})$.
   Consequentially, $\mathscr{d}\in lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}})\tilde{\bigcap}lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{N}})$.
   Hence, $lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}}\tilde{\bigcap}\mathscr{S}_{\mathscr{N}})\tilde{\subseteq}lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}})\tilde{\bigcap}lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{N}})$
3. The proof follows from Theorem (3.16).
4. Since $\mathscr{S}_{\phi}$ and $\mathscr{S}_{\tilde{\mathscr{E}}}$ are supra soft $\mathscr{v}-$ open sets, then by Theorem (2.17), the results follows.

Proposition (2.19).
Let $\mathscr{X}$ be a universal set and $\mathscr{E}$ is a set of parameters with respect to $\mathscr{X}$ and let $\{\mathfrak{F}_{\kappa}\}_{\kappa\in J}$, $\kappa\geq 2$, be a collection of supra soft topologies on $\mathscr{S}_{\tilde{\mathscr{E}}}$. If $\mathscr{S}_T\in\mathfrak{F}_{\kappa}$ for all κ∈J, then

$$lnt^{\mathfrak{F}_{\kappa}}(\mathscr{S}_{\mathrm{T}})=lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathrm{T}})=\mathscr{S}_{\mathrm{T}}.$$

Proposition (2.20).

If $\mathscr{S}_{\mathrm{T}}\in\bigcap_{\kappa\in J}\mathfrak{F}_{\kappa}$. Then $lnt^{\bigcap_{\kappa\in J}\mathfrak{F}_{\kappa}}(\mathscr{S}_{\mathrm{T}})=lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathrm{T}})$.

Proof. The result is follows from Proposition (2.19).

Example (2.21). Let $\mathscr{X}=\{u_1,u_2,u_3\}$ and $\mathscr{E}=\{e_1,e_2,e_3\}$. Define $\mathfrak{F}_1$, $\mathfrak{F}_2$ as follows:

$$\mathfrak{F}_1=\left\{\begin{matrix}\mathscr{S}_{\Phi},\mathscr{S}_{\tilde{\mathscr{E}}},\{(e_1,\{u_1\}),(e_2,\{u_2\}),(e_3,\{u_3\})\},\\ \{(e_1,\{u_1,u_2\}),(e_2,\{u_2,u_3\}),(e_3,\{u_2,u_3\})\}\end{matrix}\right\}$$

and

$$\mathfrak{F}_2=\left\{\begin{matrix}\mathscr{S}_{\Phi},\mathscr{S}_{\tilde{\mathscr{E}}},\{(e_1,\{u_3\}),(e_2,\{u_1\}),(e_3,\{u_2\})\},\\ \{(e_1,\{u_1,u_2\}),(e_2,\{u_2,u_3\}),(e_3,\{u_2,u_3\})\}\end{matrix}\right\}$$

Then $(\mathscr{X},\mathfrak{F}_1,\mathscr{E})$, $(\mathscr{X},\mathfrak{F}_2,\mathscr{E})$ are supra soft topological spaces. Now,

$$\mathfrak{F}_1\bigcap\mathfrak{F}_2=\{\mathscr{S}_{\Phi},\mathscr{S}_{\tilde{\mathscr{E}}},\{(e_1,\{u_1,u_2\}\times),(e_2,\{u_2,u_3\}),(e_3,\{u_2,u_3\})\}\}.$$

Consider:

$$\mathscr{S}_{\mathscr{V}_1}=\{(e_1,\{u_1,u_2\}),(e_2,\{u_2,u_3\}),(e_3,\{u_2,u_3\})\}$$

$$\mathscr{S}_{\mathscr{V}_2}=\{(e_1,\mathscr{X}),(e_2,\{u_2,u_3\}),(e_3,\{u_2,u_3\})\}$$

$$\mathscr{S}_{\mathscr{V}_3}=\{(e_1,\{u_1,u_2\}),(e_2,\mathscr{X}),(e_3,\{u_2,u_3\})\}$$

$$\mathscr{S}_{\mathscr{V}_4}=\{(e_1,\{u_1,u_2\}),(e_2,\{u_2,u_3\}),(e_3,\mathscr{X})\}$$

$$\mathscr{S}_{\mathscr{V}_5}=\{(e_1,\mathscr{X}),(e_2,\mathscr{X}),(e_3,\{u_2,u_3\})\}$$

$$\mathscr{S}_{\mathscr{V}_6}=\{(e_1,\mathscr{X}),(e_2,\{u_2,u_3\}),(e_3,\mathscr{X})\}$$

$$\mathscr{S}_{\mathscr{V}_7}=\{(e_1,\{u_1,u_2\}),(e_2,\mathscr{X}),(e_3,\mathscr{X})\}$$

Now, $\mathscr{S}_{\mathscr{V}_1}\in\bigcap_{\kappa=1}^{3}\mathfrak{F}_{\kappa}$ and $\mathscr{S}_{\mathscr{V}_1}\tilde{\subseteq}\mathscr{S}_{\mathscr{V}_i}$ *for all* $i=1,2,\ldots,7$, then $\mathscr{S}_{\mathscr{V}_i}$ *for all* $i=1,2,\ldots,7$ *are supra soft* $\mathscr{v}-$ *open.*

So, we have:

$$\mathcal{S}\upsilon O_{\mathcal{S}_{\tilde{\mathcal{E}}}} = \{\mathcal{S}_{\Phi}, \mathcal{S}_{\tilde{\mathcal{E}}}, \mathcal{S}_{\mathcal{V}_1}, \mathcal{S}_{\mathcal{V}_2}, \mathcal{S}_{\mathcal{V}_3}, \mathcal{S}_{\mathcal{V}_4}, \mathcal{S}_{\mathcal{V}_5}, \mathcal{S}_{\mathcal{V}_6}, \mathcal{S}_{\mathcal{V}_7}\}.$$

Now, $\mathcal{S}_{\mathcal{V}_1} \in \mathfrak{F}_{\kappa}$, for $\kappa = 1, 2$, then $\mathcal{S}_{\mathcal{V}_1} \in \mathfrak{F}_1 \bigcap \mathfrak{F}_2$ and
$lnt^{\mathfrak{F}_{\kappa}}(\mathcal{S}_{\mathcal{V}_1}) = \mathcal{S}_{\mathcal{V}_1}$ and $lnt^{\mathfrak{F}_1 \bigcap \mathfrak{F}_2}(\mathcal{S}_{\mathcal{V}_1}) = \mathcal{S}_{\mathcal{V}_1}$.
Also, $lnt_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{V}_1}) = \mathcal{S}_{\mathcal{V}_1}$. This example verifies Proposition (2.19) and Proposition (2.20).

Definition (2.22). Let $(\mathcal{X}, \mathcal{S}\upsilon O_{\mathcal{S}_{\tilde{\mathcal{E}}}}, \mathcal{E})$ be a supra soft $\upsilon-$ space and $\mathcal{S}_{\mathcal{M}} \tilde{\subseteq} \mathcal{S}_{\tilde{\mathcal{E}}}$. A point $d \in \mathcal{S}_{\mathcal{M}}$ is called an supra soft $\upsilon-$ exterior point of $\mathcal{S}_{\mathcal{M}}$ if there is $\mathcal{S}_{\mathcal{V}} \in \mathcal{S}\upsilon O_{\mathcal{S}_{\tilde{\mathcal{E}}}}$ such that $\in \mathcal{S}_{\mathcal{V}} \tilde{\subseteq} \mathcal{S}_{\mathcal{M}}{}^{c}$.

Definition (2.23). Suppose $(\mathcal{X}, \mathcal{S}\upsilon O_{\mathcal{S}_{\tilde{\mathcal{E}}}}, \mathcal{E})$ be a supra soft $\upsilon-$ space and $\mathcal{S}_{\mathcal{M}} \tilde{\subseteq} \mathcal{S}_{\tilde{\mathcal{E}}}$. The set of all supra soft $\upsilon-$ exterior point of $\mathcal{S}_{\mathcal{M}}$ is known as the supra soft $\upsilon-$ exterior of $\mathcal{S}_{\mathcal{M}}$ and is denoted by $ext_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}})$.

Theorem (2.24). Let $(\mathcal{X}, \mathcal{S}\upsilon O_{\mathcal{S}_{\tilde{\mathcal{E}}}}, \mathcal{E})$ be a supra soft $\upsilon-$ space and $\mathcal{S}_{\mathcal{M}} \tilde{\subseteq} \mathcal{S}_{\tilde{\mathcal{E}}}$. Then $ext_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}) = lnt_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}{}^{c})$.

Proof. Suppose that $d \in ext_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}})$, then d is an supra soft $\upsilon-$ exterior point of $\mathcal{S}_{\mathcal{M}}$, so there is $\mathcal{S}_{\mathcal{V}} \in \mathcal{S}\upsilon O_{\mathcal{S}_{\tilde{\mathcal{E}}}}$ such that $d \in \mathcal{S}_{\mathcal{V}} \tilde{\subseteq} \mathcal{S}_{\mathcal{M}}{}^{c}$, that is d is a supra soft $\upsilon-$ interior point of $\mathcal{S}_{\mathcal{M}}{}^{c}$, hence $d \in lnt_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}{}^{c})$, thus $ext_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}) \tilde{\subseteq} lnt_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}{}^{c})$.
Assume $d \in lnt_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}{}^{c})$, then d is a supra soft $\upsilon-$ interior point of $\mathcal{S}_{\mathcal{M}}{}^{c}$, hence there is $\mathcal{S}_{\mathcal{V}} \in \mathcal{S}\upsilon O_{\mathcal{S}_{\tilde{\mathcal{E}}}}$ such that $d \in \mathcal{S}_{\mathcal{V}} \tilde{\subseteq} \mathcal{S}_{\mathcal{M}}{}^{c}$, thus by the definition of the supra soft $\upsilon-$ exterior we have d is a supra soft $\upsilon-$ exterior point of $\mathcal{S}_{\mathcal{M}}$, hence $d \in ext_{\upsilon}(\mathcal{S}_{\mathcal{M}})$. Therefore, $ext_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}) \tilde{\supseteq} lnt_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}{}^{c})$.
Hence, $ext_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}) = lnt_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}{}^{c})$.

Theorem (2.25). If $\mathcal{S}_{\mathcal{M}} \tilde{\subseteq} \mathcal{S}_{\tilde{\mathcal{E}}}$. Then $ext_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}{}^{c}) = lnt_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}})$.

Proof. Assume that $d \in ext_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}{}^{c})$, then d is an supra soft $\upsilon-$ exterior point of $\mathcal{S}_{\mathcal{M}}{}^{c}$, so there is $\mathcal{S}_{\mathcal{V}} \in \mathcal{S}\upsilon O_{\mathcal{S}_{\tilde{\mathcal{E}}}}$ such that $d \in \mathcal{S}_{\mathcal{V}} \tilde{\subseteq} (\mathcal{S}_{\mathcal{M}}{}^{c})^{c}$, but $\mathcal{S}_{\mathcal{M}} = (\mathcal{S}_{\mathcal{M}}{}^{c})^{c}$ which implies that d is an supra soft $\upsilon-$ interior point of $\mathcal{S}_{\mathcal{M}}$, hence $d \in lnt_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}})$, thus $ext_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}{}^{c}) \subseteq lnt_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}})$.
Let $d \in lnt_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}})$, then d is an supra soft $\upsilon-$ interior point of $\mathcal{S}_{\mathcal{M}}$, hence there is $\mathcal{S}_{\mathcal{V}} \in \mathcal{S}\upsilon O_{\mathcal{S}_{\tilde{\mathcal{E}}}}$ such that $d \in \mathcal{S}_{\mathcal{V}} \tilde{\subseteq} \mathcal{S}_{\mathcal{M}}$. Now, $\mathcal{S}_{\mathcal{M}} = (\mathcal{S}_{\mathcal{M}}{}^{c})^{c}$, then $d \in \mathcal{S}_{\mathcal{V}} \tilde{\subseteq} (\mathcal{S}_{\mathcal{M}}{}^{c})^{c}$ where $\mathcal{S}_{\mathcal{V}} \in \mathcal{S}\upsilon O_{\mathcal{S}_{\tilde{\mathcal{E}}}}$, thus by the definition of supra soft $\upsilon-$ exterior we have d is a supra soft $\upsilon-$ exterior point of $\mathcal{S}_{\mathcal{M}}{}^{c}$, thus $d \in ext_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}{}^{c})$. Which implies that $ext_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}{}^{c}) \tilde{\supseteq} lnt_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}})$. Therefore, $ext_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}{}^{c}) = lnt_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}})$.

Theorem (2.26). Assume $\mathcal{S}_{\mathcal{M}} \tilde{\subseteq} \mathcal{S}_{\tilde{\mathcal{E}}}$. Then $ext_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}) = (cl_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}))^{c}$.

Proof. Since
$cl_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}) = \widetilde{\bigcap_{i \in I}} \{\mathcal{S}_{\mathcal{V}_i} : \mathcal{S}_{\mathcal{V}_i} \textit{ is supra soft } \upsilon - \textit{closed that contains } \mathcal{S}_{\mathcal{M}}\}$,
then $(cl_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}))^{c} = (\widetilde{\bigcap_{i \in I}} \{\mathcal{S}_{\mathcal{V}_i} : \mathcal{S}_{\mathcal{V}_i} \mathcal{S}_{\mathcal{V}_i} \textit{is supra soft } \upsilon - \textit{closed and } \mathcal{S}_{\mathcal{M}} \tilde{\subseteq} \mathcal{S}_{\mathcal{V}_i}\ \forall\ i \in I\})^{c}$
Now, $\mathcal{S}_{\mathcal{V}_i}$ is supra soft $\upsilon-$ closed and $\mathcal{S}_{\mathcal{M}} \tilde{\subseteq} \mathcal{S}_{\mathcal{V}_i}\ \forall\ i \in$ I, then $\mathcal{S}_{\mathcal{V}_i}{}^{c}$ is a supra soft $\upsilon-$ open and $\mathcal{S}_{\mathcal{V}_i}{}^{c} \tilde{\subseteq} \mathcal{S}_{\mathcal{M}}{}^{c}\ \forall\ i \in$ I. Hence by De-Morgan Laws we have:

$$(cl_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}))^{c} = \widetilde{\bigcup_{i \in \mathrm{I}}} \left\{ \begin{array}{c} \mathcal{S}_{\mathcal{V}_i}{}^{c} : \mathcal{S}_{\mathcal{V}_i}{}^{c} \text{ is supra soft } \upsilon - \text{open} \\ \text{and } \mathcal{S}_{\mathcal{V}_i}{}^{c} \tilde{\subseteq} \mathcal{S}_{\mathcal{M}}{}^{c}\ \forall i \in \mathrm{I} \end{array} \right\}$$

But,
$$ln\, t_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}{}^{c}) = \widetilde{\bigcup_{i \in I}} \left\{ \begin{array}{c} \mathcal{S}_{\mathcal{V}_i}{}^{c} : \mathcal{S}_{\mathcal{V}_i}{}^{c} \textit{ is supra soft } \upsilon - \textit{open} \\ \textit{and } \mathcal{S}_{\mathcal{V}_i}{}^{c} \tilde{\subseteq} \mathcal{S}_{\mathcal{M}}{}^{c}\ \forall i \in I \end{array} \right\},$$
thus $(cl_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}))^{c} = lnt_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}{}^{c})$. But from Theorem (2.24), we have
$ext_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}) = lnt_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}{}^{c})$. Hence $ext_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}) = (cl_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}))^{c}$.

Corollary (2.27). Suppose $\mathcal{S}_{\mathcal{M}} \tilde{\subseteq} \mathcal{S}_{\tilde{\mathcal{E}}}$. Then $cl_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}) = (ext_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}))^{c}$.

Proof. The result follows by Theorem (2.26).

Proposition (2.28). If $\mathcal{S}_{\mathcal{M}} \tilde{\subseteq} \mathcal{S}_{\tilde{\mathcal{E}}}$. Then $(cl_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}))^{c} = lnt_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}{}^{c})$.

Proof. The result direct by Theorem (2.24) and Theorem (2.26).

3. Conclusions

The main results of this work are:

1 $cl_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{V}})$ is the smallest supra soft $\upsilon-$ closed set that contain $\mathcal{S}_{\mathcal{V}}$.
2 $cl_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{V}}) = \mathcal{S}_{\mathcal{V}} \tilde{\bigcup} D_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{V}})$ for any $\mathcal{S}_{\mathcal{V}} \tilde{\subseteq} \mathcal{S}_{\tilde{\mathcal{E}}}$.
3 $lnt_{\mathcal{S}_\upsilon}(\mathcal{S}_{\mathcal{M}}) = \widetilde{\bigcup_{i \in I}} \left\{ \begin{array}{c} \mathcal{S}_{\mathcal{V}_i} : \mathcal{S}_{\mathcal{V}_i} \textit{ is supra soft } \upsilon - \textit{open} \\ \textit{which contained in } \mathcal{S}_{\mathcal{M}} \end{array} \right\}$.

4 $\mathscr{S}_{\mathscr{M}}$ is a supra soft $\mathscr{v}-$ openif and only if $lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}}) = \mathscr{S}_{\mathscr{M}}$.
5 $ext_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}}) = lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}}{}^c)$.
6 If $\mathscr{S}_{\mathscr{M}} \tilde{\subseteq} \mathscr{S}_{\tilde{\mathscr{E}}}$. Then $ext_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}}{}^c) = lnt_{\mathscr{S}_v}(\mathscr{S}_{\mathscr{M}})$.

With the ideas presented in this thesis, the following are some ideas and suggestions problems for the future works:

1. Studying some other concepts in supra soft $\mathscr{v}-$ space such as pre-supra soft $\mathscr{v}-$ open, semi-supra soft $\mathscr{v}-$ open, regular-supra soft $\mathscr{v}-$ open, β- supra soft $\mathscr{v}-$ open, α- supra soft $\mathscr{v}-$ open and trying to investigating their relationship.
2. Introducing the concept of supra soft $\mathscr{v}-$ open in fuzzy sets.
3. Studying the continuity, compactly and connectivity in supra soft $\mathscr{v}-$ space.
4. Planning to be introduced the separation axioms in supra soft $\mathscr{v}-$ space.

Author contribution

Luma S. Abdalbaqi: conceived of the presented and plan idea of this work and introduced the definition of supra soft space, the examples and conclusion.

Yasmin A. Hamid: developed the theory and performed the computations and the results of this paper.

All authors written the introduction and references and discussed the results and contributed to the final manuscript.

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