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REVIEW

Sensitivity Analysis of Prior Distributions in Regression Model Estimation

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Abstract

Bayesian inferences depend solely on specification and accuracy of likelihoods and prior distributions of the observed data. The research delved into Bayesian estimation method of regression models to reduce the impact of some of the problems, posed by convectional method of estimating regression models, such as handling complex models, availability of small sample sizes and inclusion of background information in the estimation procedure. Posterior distributions are based on prior distributions and the data accuracy, which is the fundamental principles of Bayesian statistics to produce accurate final model estimates. Sensitivity analysis is an essential part of mathematical model validation in obtaining a robust inference. Prior sensitivity analysis was examined in regression model, via Bayesian regression and Bayesian quantile regression analysis; results obtained across the sensitivity analysis were compared using RMSE and BIAS statistic as model performance evaluation. Empirical studies using Nigeria Economic variables were employed to analyze the variation in prior sensitivity. Different hyper parameters of the priors were used to check for sensitivity of the prior distributions, it was ascertained that Bayesian method under the frame work of regression quantiles performs well with small variance and sample size than Bayesian regression methods.

Keywords: Bayesian inferences, Prior sensitivity, Posterior distributions and regression models

1. Introduction

Regression analysis unveils the relationship between a response variable and predictor variables. Researchers are inquisitive of the behavior of a dependent variable y_t given the information contained in a set of explanatory variables x_t. However, performing a regression does not automatically infer a reliable relationship between the variables but rather, selecting an estimator that gives a precise and reliable parameter estimate [1]

Bayesian regression analysis is a statistical paradigm that answers research questions about unknown parameters using probability assertions; it provides appropriate theory for accommodating uncertainty in model selection [2,3]. Bayesian models is about integration of posterior estimates, measures of uncertainty, eliminate nuisance variables or missing data and averages models to compute predictions or perform the model comparison [4]. Accounting for uncertainty is paramount to Bayesian analysis, as the computations associated with most common tasks such as estimations, predictions, evaluation of hypothesis are typically integrations in nature. In some circumstances, it is possible to carry out such integration exactly either, by taking advantage of conjugate structure in the prior-likelihood or by using dynamic programming when the dependencies between random variables are appropriately simple [5–9]. Bayesian inference is characterized by the derivation of a posterior from a likelihood and a prior. Posterior distribution is basically dependent on a specification and accuracy of a likelihood and a prior; robustness of Bayesian inferences depends solely on this accuracy. In Bayesian context, the so-called informative prior has a fundamental impact on posterior estimates of some important models

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such as latent growth models [10], confirmatory factor model [11], Logistic regression [12] among others, as it was explored by researchers. Sensitivity analysis, is a technique in Bayesian models used to explored the behaviors of how different priors impact posterior analysis under various specified scenarios. It plays an important role in validating inferences in Bayesian model [13] under which the prior is considered as an integral part of the model building. It is argued that the combined practice of building models using informative priors and evaluating models, using prior sensitive measure, advances knowledge of model formation in Bayesian analysis [14]. Assessing the sensitivity of the Bayesian posterior inference,]15] proposed a new class of prior distributions that accounts for uncertainty in the data sequences. The researchers derived this class of prior from initial prior distribution and the likelihood distribution. The research revealed how sensitivity analysis can be performed using a standard MCMC methods for any model with closed likelihood form [16]. introduced a practical and computationally efficient methods of sensitivity analysis by estimating properties of posteriors, resulting from power-scaling the prior or likelihood by means of importance sampling estimation procedures; a diagnostic approach that can show the presence of prior-data conflict or likelihood non-informativity. More so, the research demonstrated the workflow of sensitivity analysis on case studies of real data using models that varies in complexity, from simple linear models to Gaussian process models [17]. explored the efficacy of Bayesian and Non-Bayesian methods by estimating the effect of hypothetical exposure which is influenced by measurement error of sensitivity analysis [18]. investigated the sensitivity using two different prior specifications on parameters for selection mechanism under Bayesian nonignorable selection models. In quantifying the sensitivity of prior assumption, the deviance information criterion and the conditional predictive ordinate were used as a means of comparison for goodness of fit under the two specifications of the priors. It was shown from their research that MLE prior yields better fit than the uniform prior. Besides [19], develop a general sensitivity measure based on the Hellinger distance to assess sensitivity of the posterior distributions regarding changes that occurs in the prior distributions for the precision parameters [20]. examined Prior sensitivity analysis of logistic regression model and its robustness to outliers, applying the procedures in estimating the vulnerability to poverty and anxiety. The results of the applications showed that the L-Logistic regression models provide a better fit than the corresponding beta regression models.

Sensitivity analysis has been a major focus in Bayesian model research [21]. However, the rate of the research of analyzing the sensitivity of the prior in Posterior distribution is gradually diminishing [22,23] and empirical studies employing Bayesian methods are seldomly reported [24]. Quantile regression problem was first proposed by [25] as a semi parametric extension of the linear model to estimate rates of change in all parts of the distribution of the response variable. The Bayesian framework of quantile regressions implemented via the Markov Chain Monte Carlo method for estimating regression quantiles, provides a convenient way of incorporating uncertainty into predictive inferences [26]. Quantiles play a significant role in modeling quality of service in the service industry and also in modeling risk in the financial industry. Sensitivity analysis of quantiles has been a longstanding problem due to its volatilities. Mathematically speaking, the derivative of the value of the financial product regarding the volatility is not readily available. The breakthrough research was done by [27–29]. This research will revisit the essential part of model validation by not limiting the sensitivity analysis to linear and Bayesian model alone but extends the work to Bayesian analysis of important class of quantiles. Furthermore, this work evaluates the effect of autocorrelated error in prior sensitivity of the regression models using both informative and non-informative priors in a given scenario; using likelihood–based approach. Also, the violation of the independence of the error terms may have severe negative impact on the inference if the regression parameters are not estimated efficiently. The research examined the predictive ability of a fitted model from simulated data set that has autocorrelated errors; this work is intense in analyzing robust statistical procedures for estimating parameters of regression estimates. This research re-explored works on sensitivity analysis empirically, by comparing the effects of different priors by varying their hyperparameters on their posterior estimates using Nigeria economic data. The Sensitivities analysis in this work will provide great insight in analyzing parameter used in calibrating the economic model. Comparison will be made on different regression estimates to affirm the goodness of fit using validation statistic.

2. Material and methods

This section described the methods involved in estimating regression quantiles and Bayesian estimation of regression quantiles. It also depicts the sensitivity analysis of Bayesian methods of regression analysis via Monte Carlo algorithm.

2.1. Linear regression model

Considering:

$$y = X\beta + \varepsilon \tag{1}$$

Where y is the vector of response variable, X is matrix from predictor variable., β is the vector of parameters to be estimated and, ε is the error vector

3. Method of estimating regression quantiles

Let

$$y_t = x_t^T \beta_\tau + \varepsilon_t, \tag{2}$$

y_t is the response variable and x_t a $k \times 1$ vector of covariates for the t^{th} observation. ε_t is the error term whose distribution is restricted to have τ^{th} quantile equal to zero, that is

$$\int_{-\alpha}^{0} F_\tau(\varepsilon_t) d\varepsilon_t = \tau \tag{3}$$

$$Q_\tau(x_t) = x_t' \beta \tag{4}$$

minimizing equation (4) gives a quantile estimate β_τ which is a vector of unknown parameter of interest

$$\widehat{\beta}_\tau = \operatorname{argmin}_{\beta \in R^k} \sum_{t=1}^{n} \rho_\tau(y_t - x_t'\beta_\tau) \tag{5}$$

where the loss function ρ is defined as

$$\rho_\tau(u) = [\tau - I(u < 0)]u \tag{6}$$

Solutions to the minimization cannot be derived explicitly since the lost function is not differentiable, linear programming method in 'R' was designed to obtain quantile regression estimates for $\widehat{\beta}_\tau$.

3.1. Bayesian estimation of regression quantiles

Considering the following standard linear model:

$$y_t = \mu(x_{t-1}) + \varepsilon_t$$

However

$$\mu(x_{t-1}) = x_{t-1}'\beta \tag{7}$$

$t = 1, \ldots, n$ for a vector of coefficient β

The τ^{th} conditional quantile of y_t given x_t is denoted as

$$q_\tau(y_t | x_t) = x_t'\beta(\tau) \tag{8}$$

Implementation of Bayesian quantile regression constitutes the erection of independent asymmetric Laplace distribution with $\mu(x_t) = x_t'\beta$ as the likelihood, quantiles of interest (τ) and specifications of priors on the model parameters and by Bayes theorem the resulting posterior distribution is

$$\pi(\theta | y_t, x_t', \tau) \propto L(y_t | \theta, x_t', \tau) \pi_0(\theta) \tag{9}$$

where $\pi_0(\theta)$ is the joint prior on regression parameters.

The error term follows an independent asymmetric Laplace distribution whose density function is

$$f_\tau = \tau(1-\tau)\exp\{-\rho_\tau(u)\} \tag{10}$$

for $0 < \tau < 1$, where ρ_τ is defined as the loss function of quantile regression. The minimization of the loss function of quantile regression, $\rho_\tau(u) = u[\tau - I(u < 0)]$ is exactly equivalent to the maximization of a likelihood function of asymmetric Laplace distribution Location and scale parameter μ and σ were incorporated in density (11) respectively to obtain

$$f_\tau(u, \mu, \sigma) = \frac{\tau(1-\tau)}{\sigma} \exp\left\{ -\rho_\tau\left(\frac{u-\mu}{\sigma}\right)\right\} \tag{11}$$

The method of [30] that proposed a location scale mixture of asymmetric Laplace to build a more flexible MCMC scheme in drawing samples from posterior distribution was adopted. This leads equation (12) above to

$$f(y_t | x_t', \beta, \mu.\sigma, \nu, \tau) \propto exp\left(-\sum_{i=1}^{n} \frac{(y_t - x_t'\beta - uv_i)^2}{2\delta^2 \sigma v_i} \right) \prod \frac{1}{\sigma v_i} \tag{12}$$

where v is the scaling factor for the error terms which account for the spread of the distribution across the entire quantiles. Bayesian inference depends on prior and likelihood function, conjugate prior from Normal. Uniform, Inverse gamma and Jeffrey distributions was chosen separately. The full conditional posterior distribution of $\beta, \mu.\ \sigma, \nu$ in (14) is not of tractable form

The Gibb's sampler which is an iterative Monte Carlo scheme designed to extract conditional posterior distribution from intractable joint distribution was employed for the estimation steps.

3.2. Methods of bayesian regression

Posterior distribution x likelihood function
$\propto$ *prior distribution*

Posterior Distribution.

To obtain the posterior distribution of Normal likelihood function and inverse gamma prior; consider the Normal Likelihood for the observation in equation (15)

$$f\ (y\,/\,\beta, h) = \frac{h^{\frac{N}{2}}}{(2\pi)^{\frac{N}{2}}}\, e^{\left[-\frac{h}{2}\ (y-X\beta)^{1}(y-X\beta)\right]} \tag{14}$$

where $h = \frac{1}{\sigma^2}$

the inverse gamma prior defined as equation (16) below

If $x_1|\mu, \sigma^2\ \sim N(\mu, \sigma^2\ )i.i.d\ and\ \sigma^2 \sim\ IG(\alpha, \beta)$ 4

Then

$$\sigma^2\Big|x_1, x_1, x_2, \ldots x_n, IG\left(\alpha + \frac{n}{2}, \beta + \frac{1}{2}\sum (x_i - \mu)\right. \tag{15}$$

Therefore, the posterior derived with the normal likelihood distributions and gamma prior described above is given by (17) below by reparameterization in terms of the precision S

$$P(S^{\circ}|\alpha, \beta) \propto S^{\left(a+\frac{a}{2}\right)-1} \exp\left(-S\left(\beta + \frac{1}{2}\sum (x_i - \mu)\right)\right) \tag{16}$$

3.2.1. Normal—normal

Considering the posterior distribution of Normal likelihood function and normal prior derived as follows;

$$f\ (\mu|\underline{x})\ \ \alpha f(\mu) f(\underline{x}\,|\mu) = f\ (\mu) f(x_1\,|\mu) f\ (x_2|\mu) \ldots f\ (x_n|\mu)$$

$$= \frac{1}{\sqrt{2\pi\sigma_0{}^2}} exp\left[\frac{-(\mu-\mu_0)^2}{2\sigma_0{}^2}\right] \times \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} exp\left[\frac{-(x_1-\mu)^2}{2\sigma^2}\right] \tag{17}$$

$$= \frac{1}{(2\pi)^{\left(\frac{n+2}{2}\right)} \sqrt{\sigma_0{}^2\sigma^{2n}}} exp\left[\frac{-\mu^2\ + 2\mu\mu_{0-\mu_0{}^2}}{2\sigma_0{}^2} - \sum_{i=0}^{n} \frac{x_1{}^2 - 2\mu x_1 + \mu^2}{2\sigma^2}\right] \tag{18}$$

$$= exp\left[-\frac{\left(\mu - \frac{\mu_0\sigma^2 + \sum_{i=0}^{n} \sigma_0{}^2 x_1}{\sigma^2 + n\sigma_0{}^2}\right)^2}{2\frac{\sigma_0{}^2\sigma^2}{\sigma^2+n\sigma_0{}^2}}\right] \tag{19}$$

the posterior distribution follows a normal distribution described as (21) below;

$$\mu|\underline{x}\ \sim\ N\left(\mu_1, \sigma_1{}^2\right) \tag{20}$$

where

$$\sigma_1{}^2 \frac{\sigma^2\,\sigma_0{}^2}{\sigma^2 + n\sigma_0{}^2} = \frac{1}{\sigma_0{}^{-2} + n\sigma^{-2}} \tag{21}$$

and

$$\mu_1 = \sigma_1{}^2\left(\mu_0\ \sigma_0{}^{-2} + \sum_{i=0}^{n} x_1\ \sigma^{-2}\right) \tag{22}$$

3.2.2. Normal uniform

We assign a uniform prior distribution for μ and use a normal likelihood function for the observed n measured (x_i). the posterior distribution given by

$$f(\mu) = \frac{1}{(2\pi\sigma^2)^{x/2}} exp\left(-\frac{1}{2\sigma^2} \sum_{i=0}^{x} (x_1 - \mu)^2\right) \tag{23}$$

Taking logs:

$$L(\mu)_{=-}\ n/2_{\ loge}\ (2\pi) - n\ log_e\ [\sigma]\ - \frac{1}{2\sigma^2} \sum_{i=0}^{x} (x_1 - \mu)^2 \tag{24}$$

Since S is known

$$L(s\mu) = k\ -\ \frac{1}{2\sigma^2} \sum_{i=0}^{x} (x_1 - \mu)^2 \tag{25}$$

where k is some constant. Differentiating twice we get.

$$\frac{\partial L(\mu)}{\partial\mu} = \frac{1}{\sigma^2}\left(\sum_{i=0}^{x} x_1 - \mu_n\right) \tag{26}$$

μ_0 is the average of the data value $\overline{X}$. A taylor series expansion of this function about μ0 gives

$$L(\mu) = L(\mu_0) + \frac{\partial^2 L(\mu)}{\partial\mu^2}\ \frac{(\mu-\mu_0)^2}{2} = \frac{-n}{\sigma^2}\ \frac{(\mu-\mu_0)^2}{2} \tag{27}$$

$$f(\mu) = k\,exp\left(\frac{(\mu-\overline{x})^2}{2\left(\frac{\sigma}{\sqrt{n}}\right)^2}\right) \tag{28}$$

Where k is a normalizing constant in comparison with the probability density function for the normal distribution, shows that this is normally density function with mean $\overline{X}$ and standard deviation $\sigma/\sqrt{n}$

$$\mu = N\left(\overline{x}, \sigma/\sqrt{n}\right) \tag{29}$$

3.2.3. Normal Jeffrey

Considering a Normal likelihood and Jeffery prior, a conjugate posterior is derived as;
Jeffery's prior is $[\mu] = C\sqrt{\frac{n}{\sigma^2}}$, which is still a constant (32)

Noting that the likelihood follows a normal distribution as;

$$P[\overline{X}/\mu] = \frac{1}{\sqrt{2\pi\frac{\sigma^2}{n}}} exp\left[\frac{(\overline{X}-\mu)^2}{2\frac{\sigma^2}{n}}\right] \tag{31}$$

Equations (32) and (33) gives a posterior distribution in equation 34

$$P[\overline{X}/\mu] = \frac{C\sqrt{\frac{n}{\sigma^2}}}{\sqrt{2\pi\frac{\sigma^2}{n}}} exp\left[\frac{(\overline{X}-\mu)^2}{2\frac{\sigma^2}{n}}\right] \tag{32}$$

Note that all of the stuff outside of the exponential is just a constant, so examining only the exponential, we can see that the expression is normal.

So the posterior distribution is

$$\mu \Big/ \overline{X} \sim N\left(\overline{X}, \frac{\sigma^2}{N}\right) \tag{33}$$

Based on the conditional posterior densities of β, σ^2 and μ in equation which are not analytically amenable MCMC computation method was employed to draw observations from the posterior.

3.2.4. Gibb's sampling algorithms

Starting from initial value of $\beta^{(0)}$, $\sigma^{2(0)}$ and $\mu^{(0)}$, MCMC algorithm in 6 blocks was implemented through the following steps

Step 1: Update coefficient of β from Normal posterior distribution. Sample β from full conditional posteriors. $\beta/\sigma^2, X_t \sim N(\beta, \beta)$.

Step 2: Update $\sigma^{(k)}$ from the Gamma distribution G (a,b) by drawing samples from conditional posterior distribution using $f\ (\sigma^2/X_t \sim\ G(a,b))$

Step 3: Update U^k from the posterior density $f(\mu) = \frac{1}{(2\pi\sigma^2)^{x2}} exp\left(\frac{1}{2\sigma^2}\sum_{n-1}^{n}(x_1-\mu)^2\right)$

Since the precision is known, samples were drawn from

$$P(\mu/\sigma^2, X_{1,}\ldots\ldots\ldots, X_t)$$

Step 4: Update $\mu^{(k)}$ from full Normal Jeffrey posterior. Sample were drawn from full conditional posterior $P\left(\mu/\overline{X} \sim N\left(\overline{X}, \frac{\sigma^2}{\mu}\right)\right.$

Step 5: From the Gibb's samples discard an initial number of 20000 generates as being unrepresentative as burn in and average the remaining to produce an estimate of posterior mean and standard deviation. The Monte Carlo study considered the small sample size to evaluate the effect of sample size in sensitivity of both informative and non-informative priors chosen.

3.2.5. Sensitivity analysis with autocorrelated error

Considering the regression model with auto correlated error below

$$Y_t = X_{it}^T\beta_i + \varepsilon_t \tag{34}$$

where

$$\varepsilon_t = \sum_{j-1}^{p} p_j\varepsilon_{t-j} + u_t \tag{35}$$

For t = 1, …, n, X_{it} is the q dimensional predictors, u_t follows independently identical normal with mean 0 and variance σ^2. ρ_j, j = 1, …,p is the autocorrelation coefficient of order p which determines the dependency of the error term ε_t. Given β_i, X_{it} and u_t for $t = 1,\ldots,n$, a data set for Y_t from the model in equation (36) was obtained, where $\beta_i, i = 1, \ldots, 4$ assumed 0.1,1.01,-0.06 and 1.1 respectively, the research considered 0.6 as the autocorrelation coefficient satisfying the conditions of autoregressive of order 1. The values of $(\rho, \sigma_{\varepsilon_t}^2)$ are (0.6,0.64) were used to ensure error variance of unity. The simulation study considered the small (n = 25), and large sample (n = 300) size to evaluate the effect of autocorrelated error in prior sensitivity of the regression models. The conditional posteriors obtained using both informative (Normal) and non-informative priors (uniform)were non-standard form, Gibbs sampling algorithm was thus incorporated to draw the MCMC iterates for each parameter. of the estimates, the Bayesian analysis of regression models with serially corelated errors was also extended to lower, medium and upper quantiles. The models fit was validated via root mean square error of the parameter estimates.

3.2.6. Empirical study

The intercept and the coefficient of the predictors were assigned priors and the likelihood functions defined in the methodology above were employed, the sensitivity of the priors was done to see the impact of the various priors on the posterior means by varying the hyperparameters of the priors. The Gibbs sampler ran for 120,000 replications and discard the first 20,000 as burn in period. As it is standard in quantile regression, the method was applied separately for each τ. MCMC sampling was carried out in R (R development Core Team 2022), the sensitivity of prior used were validated using RMSE and BIAS, the data set from Nigeria CBN bulletin which comprised of the Real Gross 1Domestic Product (RGDP) per capital (Y), money supply (x_1), foreign direct investment (x_2), unemployment (x_3), and non-oil export (x_6) was used using the model in equation

$$y_t = \beta_1 x_t + \beta_2 x_t + \beta_3 x_t + \beta_4 x_t + u_t \tag{36}$$

Model comparison was done between the frequentist and Bayesian methods. MCMC diagnostics indicates convergence of the Gibb's sampler. Further, the effect of these macroeconomic variable on Nigeria GDP was considered.

4. Results and discussions

This section presented the empirical results and discussions of the sensitivity analysis of the regression models described above.

Table 1 above is the empirical estimates of Ordinary Least Square regression models, posterior

Table 1. Regression models estimates.

Coefficient	OLS	Prior	Bayesian Estimates	Bayesian Quantiles Estimates					
				0.05	0.25	0.5	0.65	0.8	0.95
Intercept	663.358	NORMAL	662.96	558.12	612.4	661.37	652.92	**634.26**	**633.87**
		UNIFORM	661.85	561.34	613.34	664.64	656.35	633.23	637.34
		JEFF	618.91	**632.17**	**629.56**	616.29	633.29	639.25	642.23
		GAMMA	665.38	649.73	605.11	618.25	597.4	672.54	**633.36**
Money Supply	4.8726	NORMAL	4.641	4.345	4.896	5.19	**4.944**	**4.174**	**3.819**
		UNIFORM	4.398	3.2905	3.734	**3.236**	**2.505**	2.631	4.22
		JEFF	4.542	**2.2567**	**2.866**	3.824	2.194	3.538	3.387
		GAMMA	4.965	5.523	5.683	4.261	4.185	**3.172**	**4.185**
Foreign Direct	1.136	NORMAL	1.158	1.1947	**1.736**	2.391	1.364	**2.013**	**2.101**
		UNIFORM	1.072	0.919	0.835	**1.19**	**1.534**	**1.395**	0.872
		JEFF	1.0153	**0.2497**	**0.829**	1.632	1.183	1.876	1.937
		GAMMA	1.208	**0.371**	**0.924**	1.926	1.69	**1.773**	**1.871**
Unemployment	−3.494	NORMAL	**−3.303**	**−2.528**	**−4.502**	−2.165	−4.922	−3.274	−2.691
		UNIFORM	−2.929	−1.791	−3.572	−0.814	−0.7397	**−0.573**	−1.031
		JEFF	−2.836	**−2.063**	−4.101	−1.469	−1.932	−0.093	−0.104
		GAMMA	−3.183	**−1.041**	**−3.362**	**−3.075**	−1.403	**−0.583**	**−0.093**
Non-Oil Export	−2.853	NORMAL	0.012	0.002	0.005	**0.002**	**0.018**	**0.007**	**0.001**
		UNIFORM	0.000	0.0002	0.0004	0.0004	0.917	0.01	0.003
		JEFF	**1.000**	**1.003**	0.087	0.066	1.104	0.853	0.749
		GAMMA	1.045	**0.900**	1.000	0.701	1.042	**0.772**	**0.853**

Table 2. Performances of the estimator.

REGRESSION MODELS	QUANTILES	PRI0R DISTRIBUTIONS							
		NORMAL		UNIFORM		JEFF		GAMMA	
		RMSE	BIAS	RMSE	BIAS	RMSE	BIAS	RMSE	BIAS
BAYESIAN MODELS		0.0089	0.0024	0.1472	0.0667	0.3127	0.0536	0.2201	0.0349
BAYESIAN QUANTILES MOBELS	0.05	0.0018	0.00048	0.0298	0.00052	0.0812	0.00095	0.0013	0.00058
	0.25	0.0061	0.00071	0.0352	0.0001	0.0694	0.00037	0.0083	0.00073
	0.5	0.0053	0.00025	0.0585	0.00024	0.0338	0.00068	0.003	0.00099
	0.65	0.0029	0.00059	0.0062	0.00037	0.0417	0.00016	0.0004	0.0009
	0.8	0.0057	0.00046	0.0559	0.00085	0.0092	0.00025	0.0095	0.00085
	0.95	0.0062	0.0006	0.0016	0.007	0.0011	0.00013	0.0053	0.00028

Table 3. Performances of OLS estimator.

MODEL	RMSE	BIAS
OLS	0.0464	0.00017

estimates of Bayesian regression models and Bayesian quantile Regression models across the entire distribution. The Bayesian quantiles regression models reports the estimates of lower quantiles, medium and upper quantiles respectively. Different priors were used in checking the sensitivity of the models. From the Table 1, the Bayesian models report shows that money supply, foreign direct investment and non −oil export has a positive effect on Gross Domestic Product while unemployment reports a negative effect on Gross Domestic Product using Normal, Uniform, Jeffery and gamma prior across the entire distribution. Posterior estimates reported in bold letters are statistically significant with 95 % credible intervals formed by 2.5th and 97.5th samples quantiles of the MCMC iterates, the significant effects were more revealed in upper tails with informative priors than the non-informative priors while the significant effect is more revealing in the lower tails using Jeffery prior. The Bayesian estimate is similar to those based on quantile regression indicating the approach is practical and established parameter uncertainty.

From the Table 2, Bayesian regression models gives minimum RMSE and BIAS with Normal prior while virtually both informative and non-informative priors under study gives smallest RMSE in Bayesian quantile regression justifying the work of Yu and Moyeed 2001 that the usage of improper prior with asymmetric Laplace likelihood can lead to a proper posterior estimate. Comparing the

Table 4. Performance of regression models sensitivity with autocorrelated error.

REGRESSION MODELS	QUANTILES	PRIOR DISTRIBUTION/Sample Size							
		NORMAL N = 25		UNIFORM N = 25		NORMAL N = 300		UNIFORM N = 300	
		N (0,5) RMSE	N (1,5) RMSE	U (0,1) RMSE	U (0,10) RMSE	N (0,1) RMSE	N (1,5) RMSE	U (0,1) RMSE	U (0,10) RMSE
BAYESIAN MODELS		0.1049	0.2217	0.0963	0.0557	0.1431	0.1066	0.1369	0.0949
BAYESIAN QUANTILES	0.05	0.0224	0.0036	0.0978	0.0326	0.0076	0.0006	0.0316	0.0076
	0.25	0.0318	0.0039	0.1139	0.0446	0.0197	0.0082	0.0735	0.0197
	0.5	0.1404	0.1046	0.0992	0.1924	0.0952	0.2096	0.1164	0.2102
	0.65	0.0432	0.0052	0.2507	0.0815	0.0446	0.0012	0.0942	0.0446
	0.8	0.0442	0.0059	0.2825	0.0836	0.0068	0.0055	0.0463	0.0068
	0.95	0.0625	0.0073	0.0295	0.0927	0.0836	0.0065	0.0964	0.0836

Table 5. Prior sensitivity.

PRIOR HYPERPARAMETERS	INTERCEPT	MONEY SUPPLY	FOREIGN DIRECT	UN-EMPLOYMENT	NON-OIL EXPORT	BIAS	RMSE
NORMAL (0,1)	664.26 (1.17)	4.828 (0.15)	1.153 (0.43)	−2.694 (0.736)	0.000 (0.02)	−0.0273	0.0266
NORMAL (0,5)	667.52 (0.42)	4.806 (1.69)	1.064 (0.15)	−2.275 (0.14)	−0.000 (1.10)	−0.1237	0.0230
NORMAL (0,10)	667.59 (0.15)	4.943 (1.56)	1.174 (0.32)	−3.196 (1.27)	−0.005 (0.00)	−0.395	0.0245
NORMAL (5,10)	666.407 (0.02)	2.119 (0.23)	1.534 (0.01)	1.376 (0.52)	0.173 (0.00)	−0.2973	0.0297
NORMAL (1,5)	660.302 (0.06)	2.404 (0.32)	3.893 (0.01)	1.047 (0.16)	0.085 (0.53)	−0.482	0.0092
UNIF (0,1)	661.064 (0.01)	4.153 (0.31)	1.172 (0.17)	−3.185 (0.25)	0.000 (0.01)	−0.0004	0.4204
UNIF (0,0.5)	660.753 (1.71)	4.138 (0.04)	1.263 (0.06)	−3.266 (0.29)	0.000 (0.6)	0.0029	0.3231
UNIF (0,1)	663.725 (0.30)	2.903 (0.02)	0.432 (0.16)	0.342 (0.03)	0.028 (0.11)	−0.005	0.7277
UNIF (0,10)	670.218 (0.06)	4.722 (1.07)	3.274 (0.18)	2.691 (0.21)	0.543 (0.00)	−0.1468	0.8851
UNIF (0,0.1)	668.792 (0.01)	2.640 (0.13)	3.743 (0.5)	3.271 (0.07)	0.593 (0.29)	0.0087	0.4354
JEFF (0,1)	661.231 (0.25)	4.812 (0.00)	1.028 (0.08)	−3.012 (0.1)	0.000 (0.02)	0.0073	0.3833
JEFF (0,1.5)	660.714 (0.04)	3.842 (1.02)	2.428 (0.11)	−1.629 (0.09)	0.000 (0.09)	0.0315	265.529
JEFF (0,2)	649.143 (0.01)	2.936 (0.27)	2.032 (0.14)	0.015 (0.06)	0.005 (0.56)	0.0098	0.1164
JEFF (0,2.5)	666.392 (0.04)	3.375 (0.20)	1.914 (0.000	0.039 (0.38)	0.000 (0.41)	−0.0182	0.5281
JEFF (0,5)	661.054 (0.93)	4.252 (0.10)	1.193 (0.26)	−2.993 (0.01)	0.000 (0.04)	−0.0229	0.1349
GAMMA (1,0.5)	661.354 (0.09)	4.752 (0.26)	1.143 (0.08)	−2.993 (0.15)	0.000 (0.21)	−0.0229	0.2887
GAMMA (1,1.5)	558.135 (0.17)	4.604 (0.24)	1.215 (0.03)	−3.572 (0.00)	0.000 (0.01)	−0.0001	0.5295
GAMM (1,3)	665.715 (0.09)	1.782 (0.18)	2.683 (0.03)	−1.395 (1.16)	0.009 (0.68)	0.0004	0.7762
GAMMA (1,5)	661.272 (0.24)	4.530 (0.17)	1.262 (0.05)	−3.793 (0.93)	0.000 (0.01)	0.0021	0.8302
GAMMA (1,10)	667.07 (1.06)	1.105 (0.83)	2.528 (1.03)	1.227 (0.65)	0.003 (0.40)	−0.5593	0.1962

frequentist approach with the Bayesian approach using Tables 2 and 3, it was revealed that the Bayesian approach produced minimal RMSE and BIAS which implies that the Bayesian approach method of estimation outperformed the frequentist approach in terms of prediction performance and model accuracy.

Table 4 below gives the root mean square errors of the parameter estimates obtained from the simulation study of prior sensitivity of regression models with autocorrelated errors using both informative and non-informative priors.

The sensitivity analysis of Bayesian techniques of regression quantiles with serially correlated error for various forms of priors produces minimal error than Bayesian regression. It is observed that when dealing with both informative and non-informative priors in any chosen scenarios, the performance of Bayesian quantile regression is not visibly affected by the autocorrelation error in the model across the entire quantiles but produces higher RMSE at the median quantiles. The result justifies the work of [1] that states that inferences are not only characterized by central locations alone but must be extended to other part of the response distribution.

Table 5 shows the empirical analysis of estimation of the Bayesian regression models with different priors. This was done to see the impact of the various priors on the posterior means by varying the hyperparameters of the priors. The summary statistics reported the posterior mean with their respective standard deviations in bracket. Estimates that are statistically significant are reported in bold numbers, the result shows that Normal, Gamma, Uniform and Jeffrey's priors are all sensitive to change in hyper parameters. The lower RMSE values using Normal priors buttress a better fit to the empirical data set and the model is successfully able to explain the variations in the estimates.

5. Summary and conclusion

Sensitivity analysis is a technique used to examine how different priors can influence a posterior analysis under different scenarios. The study dug into prior sensitivity analysis using Bayesian paradigm in Bayesian models and Bayesian quantile models. The main objectives of the study were to analyze how and to what extent, prior information can influence precision of regression models using simulated and empirical data set. The work examined the predictive accuracy of model developed with informative and non-informative prior distributions and comparative analysis was done on their posterior means. The result revealed that accuracy of the estimated models with informative prior distributions is higher in Bayesian models justifying the work of [13] while Bayesian quantile regression models still performed better with non-informative priors aligned with [15] that states improper priors can still be proper in inferences with the use of Asymmetric Laplace likelihood. The work showed that sensitivities of important ranges of quantiles can be obtained in a simple and effective way using Bayesian theory. The result still valid for smaller sample size without losing power of precisions. The research filled the vacuum in the literature by delving into examination of sensitivity of prior's analysis of Bayesian quantiles regressions.

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